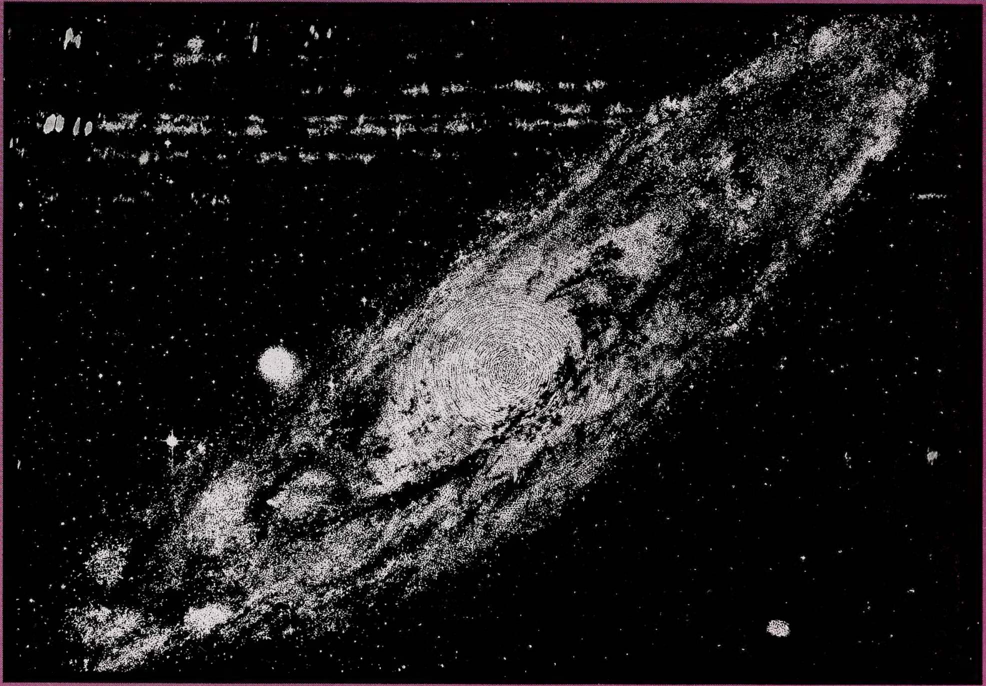
MATHEMATICS 10

UNIT 2



OPERATIONS ON POLYNOMIALS

Distance Learning

Alberta EDUCATION

Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

Mathematics 10 Student Module Unit 2 Operations on Polynomials Alberta Distance Learning Centre ISBN No. 0-7741-0745-6

General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.

- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.

- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.

- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.

- **Extensions** gives you the opportunity to take the topic one step further.

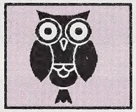
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.

- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

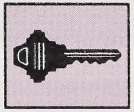
Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

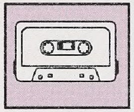
An explanation of what they mean is written beside each visual cue.



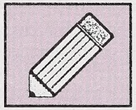
What You Already Know
- reviewing what you already know



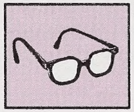
Key Idea
- flagging important ideas



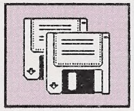
Audiotape
- learning by listening to an audiotape



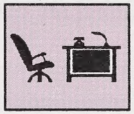
Review
- studying previous concepts



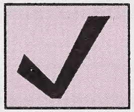
Another View
- exploring different perspectives



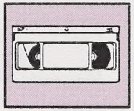
Computer Software
- learning by using computer software



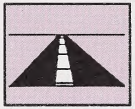
Introduction
- introducing the unit



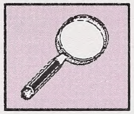
Solutions
- correcting the activities



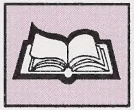
Videotape
- learning by viewing a videotape



What Lies Ahead
- previewing the unit

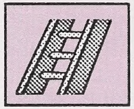


Extra Help
- providing additional study

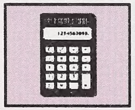


Print Pathway
- choosing a print alternative

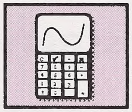
Exploring the Topic
- actively learning new concepts



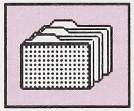
Extensions
- going on with the topic



Calculator
- using your calculator



Graphing Calculator
- using your graphing calculator



What You Have Learned
- summarizing what you have learned

Mathematics 10

Course Overview

Mathematics 10 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Operations on Polynomials	14%
Unit 3 Equations and Inequalities	10%
Unit 4 Factoring Polynomials	13%
Unit 5 Coordinate Geometry	19%
Unit 6 Systems of Equations	10%
Unit 7 Trigonometry	11%
Unit 8 Statistics	13%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Operations on Polynomials

This unit covers topics dealing with operations on polynomials. Each topic contains explanations, examples, and activities to assist you in understanding operations on polynomials. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

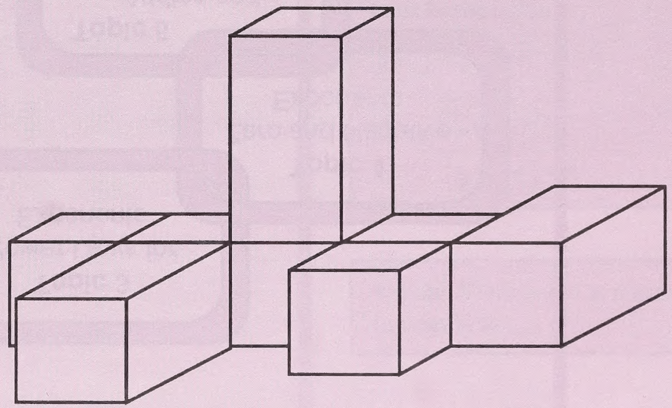
You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in **Appendix A**. In several cases there is more than one way to do the question.

Unit 2 Operations on Polynomials

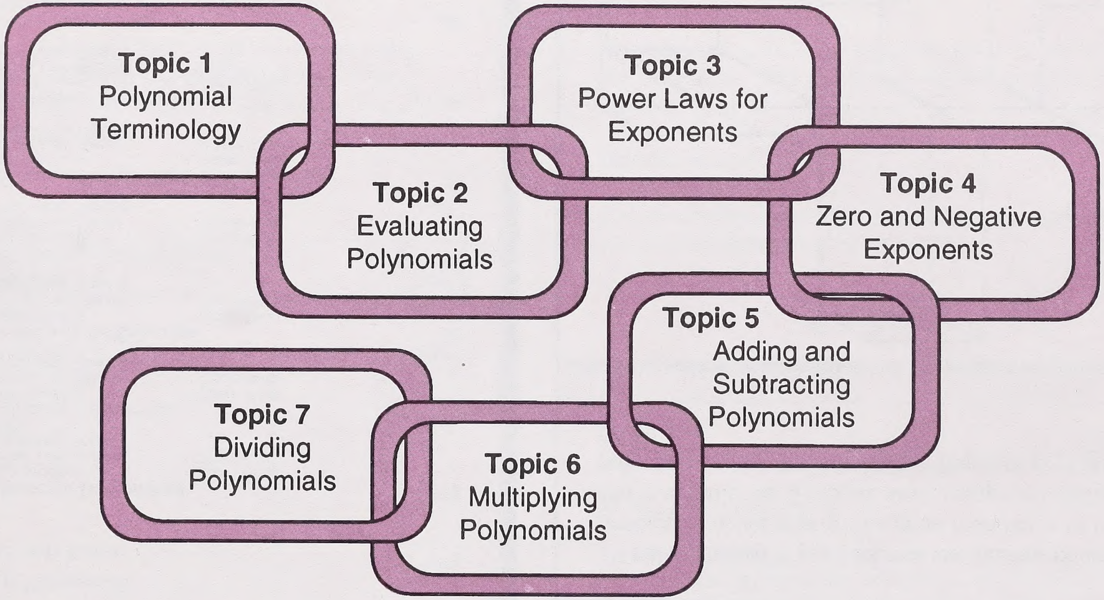
Contents at a Glance

Operations on Polynomials

To build a house, a good foundation must be poured. In mathematics you start with a good foundation of number skills and then build the algebraic walls using the building blocks of polynomials. In this unit the building blocks of polynomials will be created.



Unit 2
Operations on Polynomials



What You Already Know

Recall the following:

- You can simplify expressions with numeric bases and integral exponents using the laws of exponents.

Product Law: $\left(x^{a}\right)\times\left(x^{b}\right)=x^{a+b}$

For example,

$$\begin{aligned}\left(2^{3}\right)\left(2^{2}\right)&=2^{3+2}\\&=2^{5}\\&=32\end{aligned}$$

Power Law: $\left(x^{a}\right)^{b}=x^{ab}$

For example,

$$\begin{aligned}\left(3^{2}\right)^{4}&=3^{2\times 4}\\&=3^{8}\\&=6561\end{aligned}$$

Quotient Law: $\left(x^{a}\right)\div\left(x^{b}\right)=\dfrac{x^{a}}{x^{b}}$
$=x^{a-b}$

For example,

$$\begin{aligned}\left(6^{4}\right)\div\left(6^{2}\right)&=\frac{6^{4}}{6^{2}}\\&=6^{4-2}\\&=6^{2}\\&=36\end{aligned}$$

Power of a Product Law: $(xy)^{a}=x^{a}y^{a}$

For example,

$$\begin{aligned}(2n)^{3}&=\left(2^{3}\right)\left(n^{3}\right)\\&=8n^{3}\end{aligned}$$

Recall: The expression x^{a} is called a power, where x is the base and a is the exponent.

When using the Product Law or the Quotient Law, the bases must be the same.

$x^{a}\bullet x^{b}=x^{a+b}$

$$\begin{aligned}x^{a}\bullet y^{b}&=(xy)^{a+b}\\&\neq x^{a+b}y^{a+b}\end{aligned}$$

Recall: The symbol $\neq$ means **is not equal to.**

Power of a Quotient Law: $(x \div y)^a = \left(\frac{x}{y}\right)^a$

$$= \frac{x^a}{y^a}$$

For example,

$$(4 \div 5)^2 = \left(\frac{4}{5}\right)^2$$
$$= \frac{4^2}{5^2}$$
$$= \frac{16}{25}$$

- Remember that the only way powers can be added or subtracted is by first finding the standard name, and then finding the sum or difference.

For example,

$$4^2 + 5^2 - 2^3 = 16 + 25 - 8$$
$$= 41 - 8$$
$$= 33$$

- Rewrite large and small numbers using scientific notation.

For example,

$$93\,000\,000 = 9.3 \times 10^7$$

$$0.000\,000\,389 = 3.89 \times 10^{-7}$$

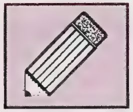
- Use the order of operations for simplifying numerical expressions.

For example,

$$\frac{2(5)+3(3)^2-1}{2^2+2} = \frac{2(5)+3(9)-1}{4+2}$$
$$= \frac{10+27-1}{4+2}$$
$$= \frac{37-1}{6}$$
$$= \frac{36}{6}$$
$$= 6$$

Now that you have looked at material studied previously, go to the **Review** to confirm your understanding of this material.

Recall: In $4^2 = 16$, the number 16 is called the standard name or the standard form.



Review

1. Simplify the following expressions by first using the exponent laws and then writing the solutions as a number in standard form. Use your calculator to solve a, b, d, e, and g.

   a. 3^4

   b. $3^2 \times 3^4$

   c. $12^6 \div 12^5$

   d. $\left(2^5\right)^2$

   e. $\left(3^2 \times 2^3\right)^3$

   f. $\dfrac{5^3}{5^2}$

   g. $\left(\dfrac{3^2}{5^3}\right)^3$

   h. $3^2 + 4 \times 3$

   i. $3^2 + 3^2 - 2^3$

   j. $4\left(15 - 3^2\right) \div 8 + 2$

2. Rewrite the following numbers using scientific notation.

   a. 210 000 000

   b. 0.000 000 060 73

3. Rewrite the following numbers in standard form.

   a. 4.50×10^6

   b. 3.0014×10^{-7}

4. Describe the situation and explain how the Power of a Product Law is used to simplify an expression.

5. Explain why scientific notation is a useful way to express numbers.

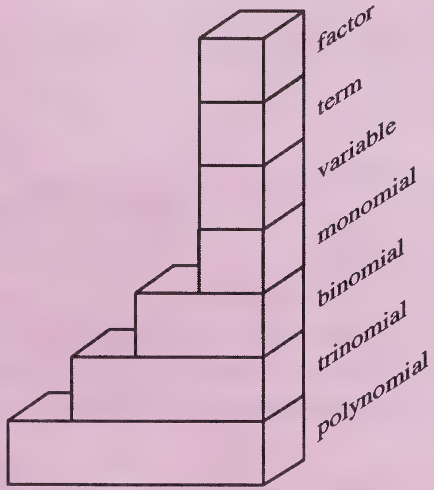
Now go to the **Review** solutions in **Appendix A**.

If you had trouble with the **Review**, go to **Mathematics 9, Module 2**.

Topic 1 Polynomial Terminology

Introduction

Blocks can be made of clay, stone, or slate. Most people want to know the composition of materials that make up their homes. Polynomial building blocks are made up of **terms**. In this topic, the terms used in polynomials will be analyzed.



What Lies Ahead

Throughout this topic you will learn to

1. identify term, variable, factor, monomial, binomial, trinomial, polynomial, numerical coefficient, literal coefficient, degree, exponent, base, and power in the study of polynomials

2. classify polynomials according to degree, number of terms, and number of variables

Now that you know what to expect, turn the page to begin your study of polynomial terminology.

Exploring Topic 1

Activity 1

Identify term, variable, factor, monomial, binomial, trinomial, polynomial, numerical coefficient, literal coefficient, degree, exponent, base, and power in the study of polynomials

The study of polynomials uses a wide range of vocabulary with which you must become familiar. A good place to start is with the language of exponents.

Exponents

In the expression 3^5, 3 is called the **base**, 5 is called the **exponent**, and the expression 3^5 is called the **power**.

An exponent is used to show how many times a given base is multiplied by itself or, in other words, how many times a base is to be used as a factor.

The expression 3^5 means that 3 is used as a factor five times.

$$3^5 = \underbrace{3\times3\times3\times3\times3}_{\text{five factors}}$$

The expression 3^5 is the **simplified form** and $3\times3\times3\times3\times3$ is the **expanded form**. The numerical value 243 is the **standard** or **evaluated form**.

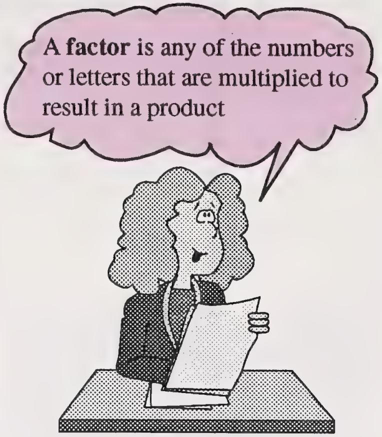
Terms

An **algebraic term** is an expression containing the product of only numbers and/or algebraic symbols. All algebraic terms have a number as one of the factors. This numerical factor is called the **numerical coefficient**. When a numerical coefficient is not written with the term, it is assumed to be one.

- The expression *abc* is an algebraic term that has a numerical coefficient of one.

The non-number factors of the algebraic term are called the **literal coefficients** or **variables**.

- In *abc*, *a*, *b*, and *c* are literal coefficients or variables.



When an algebraic term has no literal coefficient, it consists of only a number. Such a number by itself is called a **constant term**.

- The number 16 has no literal coefficient and is called a constant term.

The following are more examples of algebraic terms:

- -15
  The number -15 is a constant.

- $5a^2$
  The numerical coefficient is 5. The literal coefficient is a^2.

- $-7x^4y$
  The numerical coefficient is -7. The literal coefficients are x^4 and y.

- ab^2
  The numerical coefficient is 1. The literal coefficients are a and b^2.

- $-x$
  The numerical coefficient is -1. The literal coefficient is x.

- $(4a)^2 = 16a^2$
  The numerical coefficient is 16. The literal coefficient is a^2.

- $\left(-2y^2\right)^3 = -8y^6$
  The numerical coefficient is -8. The literal coefficient is $-y^6$.

- $-(3y)^2 = -9y^2$
  The numerical coefficient is -9. The literal coefficient is y^2.

The literal coefficients can be written in any order, but it is generally accepted that the literal coefficients should be written in alphabetical order to be classified as being in simplified form.

Polynomials

A polynomial is a term or a sum of terms. In a polynomial the exponents of the literal coefficients must be whole numbers, and the literal coefficients must be in the numerator.

Examples are as follows:

- One term: x, $2x^2$, $\frac{3}{5}a$, $\frac{0.2xy^2}{3}$
- Sum of terms: $2x+3$, $ax^2+bx+c-3y^2-2$

There are special names for polynomials that have one, two, or three terms. A polynomial that has one term is called a monomial (mono means one). A binomial (bi means two) has two terms, and a trinomial (tri means three) has three terms.

Examples are as follows:

- Monomial: xy, $\frac{1}{2}x^2$, $\frac{2a}{3}$
- Binomial: $ab+xy$, $3x^2y-2y^3$
- Trinomial: x^2+2x+3

The **degree** of a term is the sum of the exponents of all of the variables in the term. The **degree of a polynomial** is the degree of the term that has the largest sum of exponents.

Examples are as follows:

- The degree of $2x^2y$ is 3 since the exponent of x^2 is 2 and the exponent of y is 1.

  $2+1=3.$

- The degree of $5x^2y^2-3x^3+6y^2$ is 4, since the degree of $5x^2y^2$ is $2+2=4$. The degree of $-3x^3$ is 3, and the degree of $6y^2$ is 2.

  The term that has the largest degree is $5x^2y^2$; thus, the degree of the whole polynomial $5x^2y^2-3x^3+6y^2$ is 4.

Polynomials should be written so that the degree of the terms are in descending order. If the polynomial has only one literal coefficient, the term with the highest degree would be written first followed by the term with the second highest degree. If the polynomials has more than one literal coefficent, the terms will first be arranged alphabetically according to the first literal coefficient (then the second or third coefficients, if necessary). For those terms that have the same literal coefficient, the term that is written first will have the largest exponent with that coefficient. The second term will have the second largest exponent and so on.

When no exponent is shown, it is understood to be 1.

Recall: The expression $3y^2-2$ is a sum because
$3y^2-2=3y^2+(-2)$.

$$\frac{2a}{3}=\frac{2}{3}a$$

Study the following examples to see how this is used.

Example 1

Arrange the following polynomials in descending order.

- $3+2d$

  Solution:

  $3+2d=2d+3$
  A term with a variable always comes before a constant term.

- $2t+3t^4+3t^2+2$

  Solution:

  $2t+3t^4+3t^2+2=3t^4+3t^2+2t+2$
  There is only one literal coefficient so the terms are written from the highest to the lowest degree.

- $4ab^2+5ab^5+3a^2$

  Solution:

  $4ab^2+5ab^5+3a^2=3a^2+5ab^5+4ab^2$

  The terms are first arranged according to the variable *a*, highest degree to lowest degree. When the *a* has the same exponent, the terms are ordered according to the variable *b*, highest degree to lowest degree.

- $3x^2y+4x^2y^2+2x^2y^3$

  Solution:

  $3x^2y+4x^2y^2+2x^2y^3=2x^2y^3+4x^2y^2+3x^2y$
  Since all of the terms have the same exponent of *x*, the terms are arranged according to the exponents of the *y* factors.

Now try the questions that follow.

Do at least two parts from each of the following questions.

1. State the degree for each term.

   a. 2a
   b. $3d^2m^3n$
   c. $36xy^3$
   d. $19m^2np^0$
   e. 11
   f. $21a^3b^2cq^2$

2. State the degree for each polynomial.

   a. $3a^2b+6ab^2+b^3$
   b. $4x^2y^3+3xy+4y+34$
   c. $16p^5q^3+12p^4q^3+9p^4q^2+8pq$
   d. $2m^4n^2+2m^3n^6+3m^2n^6+2mn^5$

3. Write each of the following in simplified form and in standard form.

a. $3\times3\times3\times3$

b. $(-4)(-4)(-4)(-4)(-4)$

c. 31×31

d. $612\times612\times612$

4. Which of the following are polynomials? For each polynomial state the numerical coefficient of x in each x-term, and state the constant term.

a. $2x^4+5x^2+3$

b. $3x^2+9x$

c. $4x^2-4x^{\frac{1}{3}}$

d. $-8x^{10}$

5. Explain why each of the following is not a polynomial.

a. $3x^2+x-\frac{3}{x}$

b. $2y^2+2y^{\frac{3}{2}}+y$

c. $5m^3+5m^{-2}+3m$

d. $7n^2+\frac{8n}{3n^2}$

6. Write the following polynomials in descending order.

a. $3mn+3m^2n+3m^2n^2$

b. st^2+2s^2t+3s

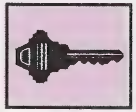
c. $6a+7b^3c+2b^2c^2$

d. x^2+2y^2+3xy

For solutions to **Activity 1**, turn to **Appendix A**, **Topic 1**.

Activity 2

Classify polynomials according to degree, number of terms, and number of variables.



The following charts list the most commonly used classifications for polynomials.

Classifications by Degree	
Name	Highest Degree
Constant	0
Linear	1
Quadratic	2
Cubic	3
Quartic	4
Fifth Degree	5

Classifications by Number of Terms	
Name	Number of Terms
Monomial	1
Binomial	2
Trinomial	3
Polynomial of four terms	4

Classifications by Number of Variables	
Name	Number of Variables
Polynomial with one variable	1
Polynomial with two variables	2
Poynomial with three variables	3

Now apply this information in the following example.

Example 2

- Find the degree of each of the following polynomials:

$3,\ 3x,\ 5x-2y+3,\ 5x^2-2y+3,\ 4x^3-5x,\ 4x^4,\ 4xy^3+34,$
$4x^2y^3+3xy+4y+34$

Solution:

Polynomial	Degree
3 no literal coefficient	The degree is 0.
$3x$ 1	The degree is 1.
$5x-2y+3$ 1 1 0	The degree is 1.
$5x^2-2y+3$ 2 1 0	The degree is 2.
$4x^3-5x$ 3 1	The degree is 3.
$4x^4$ 4	The degree is 4.
$4xy^3+34$ 1+3 0 4	The degree is 4.
$4x^2y^3+3xy+4y+34$ 2+3 1+1 1 0 5 2	The degree is 5.

- Classify each of the previous polynomials according to the number of terms in the polynomial, the degree of the polynomial, and the number of variables in the polynomial.

Solution:

Polynomial	Terms	Degree	Variables
3	monomial	0 (constant)	no variables
$3x$	monomial	1 (linear)	one variable
$5x-2y+3$	trinomial	1 (linear)	two variables
$5x^2-2y+3$	trinomial	2 (quadratic)	two variables
$4x^3-5x$	binomial	3 (cubic)	one variable
$4x^4$	monomial	4 (quartic)	one variable
$4xy^3+34$	binomial	4 (quartic)	two variables
$4x^2y^3+3xy+4y+34$	polynomial of four terms	5 (fifth degree)	two variables

Try the questions that follow.

Do any four problems in each of the following questions.

1. Classify the following polynomials according to their degree.

   a. 15
   b. $3y+3$
   c. $16x^2+8x+4$
   d. x^3-5
   e. $16x^2yz+12xy^2z+3y^3z$
   f. $22x^2+23y^2+11z^2$

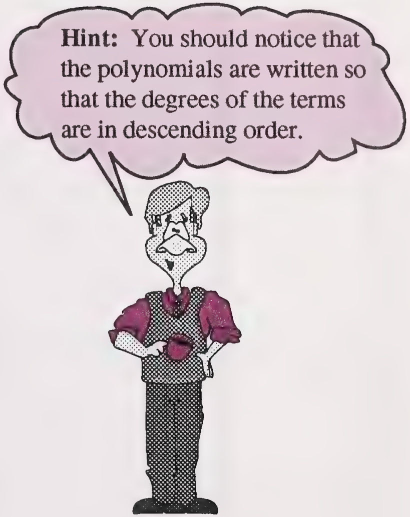
2. Classify the following polynomials according to the number of terms.

   a. 15
   b. $3y+3$
   c. $16x^2+8x+4$
   d. x^3-5
   e. $16x^2yz+12xy^2z+3y^3z$
   f. $22x^2+23y^2+11z^2$

3. Classify the following polynomials according to the number of variables.

   a. 15
   b. $3y+3$
   c. $16x^2+8x+4$
   d. x^3-5
   e. $16x^2yz+12xy^2z+3y^3z$
   f. $22x^2+23y^2+11z^2$

For solutions to **Activity 2**, turn to **Appendix A, Topic 1**.



If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

To help you understand polynomials and the terminology surrounding them, do the following question. Match Column A with Column B. Write the number of your choice in the blank space provided. The first blank is done as an example.

Column A		Column B
1. $3x^2 + 2y - 3x + 4$	5	This term is of degree 7.
2. 2^5	______	This polynomial is a binomial.
3. $5x^2y^2$	______	This polynomial is a trinomial.
4. 4^6	______	This polynomial is a monomial of degree 3.
5. $x^2y^3z^2$	______	This polynomial has four terms.
6. 2^4	______	This power has an exponent of 5.
7. $x^2 - 5x - 6$	______	This power has a base of 4.
8. $6x^2$	______	This power has a value of 16.
9. $5x^3$	______	The literal coefficient of this term is x^2y^2.
10. $2x^2 - 3x$	______	The numerical coefficient of this term is 6.
11. $17x^{-2}$	______	This expression is not a polynomial.

For solutions to **Extra Help**, turn to **Appendix A**, **Topic 1**.

Extensions

To test your knowledge of polynomials, do the following question. Match Column A with Column B. Write the number of your choice in the blank space provided.

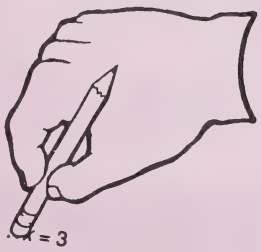
Column A	Column B	
1. $5x^4 + 2xyz$	______	This is a binomial of degree 4 in two variables.
2. $3x^3 + y^2$	______	This is a binomial of degree 3 in three variables.
3. $5xy - 3z$	______	This is a binomial of degree 4 in three variables.
4. $3x^2y - 4xy^3$	______	This is a binomial of degree 3 in two variables.
5. $5x^3 + 4yz$	______	This is a binomial of degree 2 in three variables.
6. $3xy - y$	______	This is a binomial of degree 2 in two variables.

For solutions to **Extensions**, turn to **Appendix A, Topic 1**.

Topic 2 Evaluating Polynomials

Introduction

Letters are often confusing in mathematics. Some people prefer straight number work. In this topic you are going to eliminate the letters by replacing them with numbers. Thus, you are going to calculate the value of a polynomial when specific values are assigned to the variables.



What Lies Ahead

Throughout the topic you will learn to

1. evaluate polynomials for given values of a variable

Now that you know what to expect, turn the page to begin your study of evaluating polynomials.

Exploring Topic 2

Activity 1

Evaluate a polynomial for given values of a variable.

Examine the following examples to see how a polynomial is evaluated when the variable is assigned a specific value.

Example 1

- Evaluate x^3 when $x = 3$.

  Solution:

  $$\begin{aligned} \text{When } x = 3,\ x^3 &= (3)^3 \\ &= 27 \end{aligned}$$

- Evaluate x^3 when $x = -5$.

  Solution:

  $$\begin{aligned} \text{When } x = -5,\ x^3 &= (-5)^3 \\ &= -125 \end{aligned}$$

- Evaluate x^3 when $x = \frac{2}{3}$.

  Solution:

  $$\begin{aligned} \text{When } x = \frac{2}{3},\ x^3 &= \left(\frac{2}{3}\right)^3 \\ &= \frac{8}{27} \end{aligned}$$

Sometimes there is more than one value to be substituted into a polynomial.

Example 2

Evaluate $-5a^2 + 2b$ when $a = -2$ and $b = -\frac{5}{2}$.

Solution:

$$\begin{aligned} -5a^2 + 2b &= -5(-2)^2 + 2\left(-\frac{5}{2}\right) \\ &= -5(4) - 5 \\ &= -20 - 5 \\ &= -25 \end{aligned}$$

Sometimes the question will be asked in a different way.

Remember: Watch the sign of the variable. Recall that $(-)(-)$ becomes $+$, and $(-)(-)(-)$ becomes $-$.

Hint: Whenever you replace a variable with a given value, use parentheses to show where the variable has been replaced.

For example, $x^3 = (3)^3$
$= 27$

Remember: To do the order of operations, use the **BEDMAS** rule.

Brackets
Exponents
Division, or / Multiplication } in order from left to right
Addition, or / Subtraction } in order from left to right

Example 3

If $p(x)=3x^2-5x+2$, evaluate $p\left(\frac{2}{3}\right)$.

Solution:

$$\begin{aligned} p(x)&=3x^2-5x+2 \\ p\left(\frac{2}{3}\right)&=3\left(\frac{2}{3}\right)^2-5\left(\frac{2}{3}\right)+2 \\ &=3\left(\frac{4}{9}\right)-5\left(\frac{2}{3}\right)+2 \\ &=\frac{4}{3}-\frac{10}{3}+2 \\ &=0 \end{aligned}$$

Writing algebraic expressions to represent given situations is a valuable skill in mathematics.

Example 4

Translate the following phrases about nickels and dimes saved in a jar. Let d represent the number of dimes.

- Give an expression for the number of nickels if there are twice as many nickels as dimes.

  Solution:

  $2d$

- Give the value of the dimes in cents.

  Solution:

  $10d$ cents

- Give the value of the nickles in cents.

  Solution:

  $5(2d)=10d$ cents

- Give the total value of the dimes and nickels in cents.

  Solution:

  $10d+10d=20d$ cents

- If there are six dimes, how many nickels are there?

  Solution:

  $$\begin{aligned} \text{Number of nickels}&=2d \\ &=2(6) \\ &=12 \end{aligned}$$

  There are twelve nickels in the jar.

Note that $p\left(\frac{2}{3}\right)$ **means the value of** $\mathbf{3x^2-5x+2}$ **when** $\mathbf{x=\frac{2}{3}}$**.**

- If there are eighteen nickels, what is the value of the coins in the jar?

Solution:

$$\begin{aligned}\text{Number of nickels} &= 2d\\ 18 &= 2d\\ \frac{18}{2} &= d\\ d &= 9\end{aligned}$$

$$\begin{aligned}18\text{ nickels} &= 18\times 5\\ &= 90\text{ cents}\end{aligned}$$

$$\begin{aligned}9\text{ dimes} &= 9\times 10\\ &= 90\text{ cents}\end{aligned}$$

$$\begin{aligned}\text{Total value of the coins} &= 90¢ + 90¢\\ &= 180¢\\ &= \$1.80\end{aligned}$$

The value of the coins in the jar is $1.80.

Now try the following questions.

Do at least four parts of questions 1, 2, and 3. Then do questions 4 and 5.

1. Evaluate the following expressions when $x = 4$.

   a. $x+7$

   b. $x-11$

   c. $3x-4$

   d. $16-2x$

   e. $5x^2-32$

   f. $5x^2-3x+15$

   g. x^4+3x^3+2x

   h. $3x^4+2x-3$

2. Evaluate the following expressions when $p = 2$, $q = -3$, and $r = 6$.

   a. $p-2q$

   b. $3q+r$

   c. $5r+3q$

   d. $3p-2q-5r$

   e. $3r^2-5q$

   f. $4p^2+3q^3$

   g. $q^2-3r^2+2p^3$

   h. $r^2-3q^2+4p^3$

3. If $f(y) = 3y^3 - 2y + 15$, find the following values.

   a. $f(1)$
   b. $f(4)$
   c. $f(-1)$
   d. $f(-10)$
   e. $f(0)$
   f. $f\left(\frac{5}{2}\right)$
   g. $f(-3) + f(2)$
   h. $f(-1) + 2f(0)$

4. The height of a diver above water level, in metres, is given by the equation $-4.9t^2 + 9.8t + 10$, where t is time in seconds. How far above the water is the diver after the following number of seconds?

   a. one second
   b. two seconds
   c. three seconds

5. Explain some situations in which you would want a polynomial evaluated by substituting different values for the variables.

6. Write an expression for each of the following:

   a. the number of eggs in d dozen
   b. the number of tires on k cars
   c. the number of minutes in n hours
   d. the cost in cents of p pears that cost 35¢ each
   e. the total cost in cents of b boxes of candy at 45¢ each and c chocolate bars at 60¢ each

For solutions to **Activity 1**, turn to **Appendix A, Topic 2**.

Hint: When solving problems, do the following:

- Plan a solution strategy.
- Carry out the plan.
- Look back.
- Present the solution in a sentence.

If you require help, do the Extra Help section.
If you want more challenging explorations, do the Extensions section.
} You may decide to do both.

Extra Help

Your calculator can be a useful tool in helping you evaluate polynomials.

The following examples show how to use a scientific calculator to evaluate polynomials.

Example 5

Evaluate $2x+7$ when $x=4$.

Solution:

Substitute $x=4$ in $2x+7$.
Evaluate $2\times4+7$

Enter	Display
2	2
$\boxed{\times}$	2
4	4
$\boxed{+}$	8
7	7
$\boxed{=}$	15

The x^y function may be used to evaluate powers.

4^5

Enter	Display
4	4
$\boxed{x^y}$	4
5	5
$\boxed{=}$	1024

Note: Some calculators have the $\boxed{y^x}$ key instead of the $\boxed{x^y}$ key.

If your calculator does not have an x^y function, then 4^5 would be evaluated as follows:

Enter	Display
4	4
$\boxed{\times}$	4
4	4
$\boxed{\times}$	16
4	4
$\boxed{\times}$	64
4	4
$\boxed{\times}$	256
4	4
$\boxed{=}$	1024

or

Enter	Display
4	4
$\boxed{\times}$	4
$\boxed{=}$	16
$\boxed{=}$	64
$\boxed{=}$	256
$\boxed{=}$	1024

Example 6

Evaluate $3x^2 - 7$ when $x = 4$.

Solution:

Substitute $x = 4$ in $3x^2 - 7$.

Evaluate $3(4)^2 - 7$.

Enter	Display
3	3
$\boxed{\times}$	3
4	4
$\boxed{x^2}$	16
$\boxed{-}$	48
7	7
$\boxed{=}$	41

If your calculator does not have an $\boxed{x^2}$ key, follow this method:

Enter	Display
3	3
$\boxed{\times}$	3
4	4
$\boxed{\times}$	12
4	4
$\boxed{-}$	48
7	7
$\boxed{=}$	41

Example 7

Evaluate $2x^3 - 3x^2$ when $x = 5$.

Solution:

Evaluate $2(5)^3 - 3(5)^2$.

Enter	Display
2	2
$\boxed{\times}$	2
5	5
$\boxed{x^y}$	5
3	3
$\boxed{-}$	250
3	3
$\boxed{\times}$	3
5	5
$\boxed{x^y}$	5
2	2
$\boxed{=}$	175

Example 8

Evaluate $2x^2y + 3xy^3$ when $x = -2$ and $y = -3$.

Solution:

Evaluate $2(-2)^2(-3) + 3(-2)(-3)^3$.

Enter	Display
2	2
$\boxed{\times}$	2
2	2
$\boxed{+/-}$	−2
$\boxed{x^2}$	4
$\boxed{\times}$	8
3	3
$\boxed{+/-}$	−3
$\boxed{+}$	−24
3	3
$\boxed{\times}$	3
2	2
$\boxed{+/-}$	−2
$\boxed{\times}$	−6
3	3
$\boxed{+/-}$	−3
$\boxed{x^y}$	−3
3	3
$\boxed{=}$	138

Note: The $\boxed{+/-}$ function is used to change a positive number to a negative number and vice versa.

$-2 = \boxed{2}\ \boxed{+/-}$

Evaluate the following polynomials with a scientific calculator.

1. $2x-7$ when $x=3$
2. $3x+1$ when $x=7$
3. $11-3x^3$ when $x=-2$
4. $3x^2-2x+3$ when $x=5$
5. $-3x+2x^2-7$ when $x=-3$
6. $2x^3+5x^2$ when $x=4$

For solutions to **Extra Help**, turn to **Appendix A, Topic 2**.

Note that when you attempt to evaluate an expression such as $(-3)^3$ on some scientific calculators, an error results.

Enter	Display
3	3
+/–	−3
x^y	−3
=	0

Some calculators will not accept negative number raised to a power. If your calculator is one of these, use one of the alternative methods outlined to evaluate any similar expressions.

The alternatives are as follows:

- $(-3)^3$

Enter	Display
3	3
+/–	−3
×	−3
=	9
=	−27

- Remember these two rules.

$(\text{negative number})^{\text{even power}} = \text{positive number}$

$(\text{negative number})^{\text{odd power}} = \text{negative number}$

Examples of these rules are as follows:

- $(-2)^4 = 16$
- $(-2)^5 = -32$

To evaluate $(-3)^3$, evaluate 3^3. Since the exponent is odd, the answer is -27.

Extensions

With the aid of a scientific calculator, you may evaluate any polynomial regardless of how difficult it is. If you are not familiar with evaluating polynomials with your calculator, refer to the preceding **Extra Help** section. If you are familiar with evaluating polynomials with a calculator, you may look at the following examples or go directly to the questions that follow.

Example 9

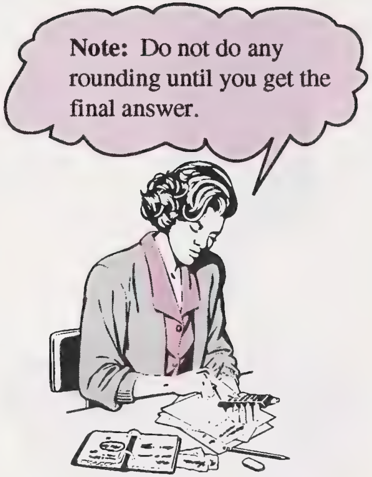
Evaluate $2x^3 - 3x^2 + 227$ when $x = -5.73$. Round your answer to two decimal places.

Solution:

$$\begin{aligned} &2x^3 - 3x^2 + 227 \\ &\quad = 2(-5.73)^3 - 3(-5.73)^2 + 227 \end{aligned}$$

Enter	Display
2	2
$\boxed{\times}$	2
5.73	5.73
$\boxed{+/-}$	−5.73
$\boxed{x^y}$	−5.73
3	3
$\boxed{-}$	−376.265034
3	3
$\boxed{\times}$	3
5.73	5.73
$\boxed{+/-}$	−5.73
$\boxed{x^2}$	32.8329
$\boxed{+}$	−474.763734
227	227
$\boxed{=}$	−247.763734

Rounded to two decimal places, the answer is -247.76.



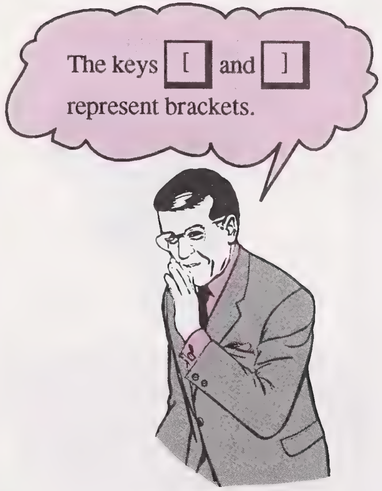
Example 10

Evaluate $5(2x-3)^3 - 2(x+2)^2$ when $x=5$.

Solution:

Evaluate $5[2(5)-3]^3 - 2(5+2)^2$.

Enter	Display
5	5
$\boxed{\times}$	5
$\boxed{[}$	0
2	2
$\boxed{\times}$	2
5	5
$\boxed{-}$	10
3	3
$\boxed{]}$	7
$\boxed{x^y}$	7
3	3
$\boxed{-}$	1715
2	2
$\boxed{\times}$	2
$\boxed{[}$	0
5	5
$\boxed{+}$	5
2	2
$\boxed{]}$	7
$\boxed{x^y}$	7
2	2
$\boxed{=}$	1617



Evaluate the following polynomials with a calculator. Round to one number after the decimal point.

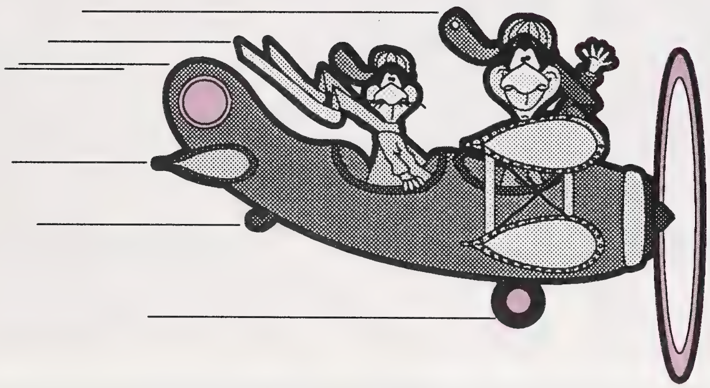
1. $2x^2 - 5x - 1$ when $x = 2.4$

2. $3x^2 - 7x + 3$ when $x = -1.7$

3. $(2x-3)^3 - x^4$ when $x = 2.3$

4. $3x^7 - 5x^6 - 10$ when $x = 0.8$

For solutions to **Extensions**, turn to **Appendix A, Topic 2**.



Remember to only round the final answer. Do not round during the steps of the calculation.

Topic 3 Power Laws for Exponents

Introduction

In building a house you might need some materials to join the walls. Rules of exponents are materials that are needed before you can put all the polynomial operations together. You will now examine this material.

What Lies Ahead

Throughout the topic you will learn to

1. use the Multiplication or Product Law of exponents for powers with literal bases and whole number exponents
2. use the Power of a Power Law for exponents
3. use the Division or Quotient Law of exponents for powers with literal bases and whole number exponents
4. use the Power of a Product Law for exponents
5. use the Power of a Quotient Law for exponents
6. verify each of the power laws for exponents

Now that you know what to expect, turn the page to begin your study of power laws for exponents.

Exploring Topic 3

Activity 1

Use the Multiplication or Product Law of exponents for powers with literal bases and whole number exponents.

Is there a short way of finding the product of 3^5 and 3^2 or a^6 and a^7?

Study the solution of these two questions and see if you can find a short-cut to do this multiplication operation.

Case 1

$$3^5 \times 3^2 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)$$
$$= 3^7$$

$$3^5 \times 3^2 = 3^{5+2}$$
$$= 3^7$$

It seems that the exponent in the product is the sum of the exponents in the factors. Now see if this is true for the second case.

Case 2

$$a^6 \times a^7 = (a \times a \times a \times a \times a \times a) \times (a \times a \times a \times a \times a \times a \times a)$$
$$= a^{13}$$

$$a^6 \times a^7 = a^{6+7}$$
$$= a^{13}$$

This case has the same results.

No matter what example you choose, you would have the same results as long as the base is the same in the factors being multiplied.

This is called the **Product Law** or the **Multiplication Law**. It can be expressed as follows using general terms.

$x^m \times x^n = x^{m+n}$ where $m, n \in W$

3^6 is a power. The 3 is the base and the 6 is the exponent.

Note: $n \in W$ means that n is a whole number.

Summarize this information into a chart.

Expanding	Number Bases	Variable Bases	Exponent Law
$2^3 \times 2^4 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ $= 2^7$	$2^3 \times 2^4 = 2^{3+4}$ $= 2^7$	$x^3 \times x^4 = x^{3+4}$ $= x^7$	$x^m \times x^n = x^{m+n}$ Product Law (Multiplication Law)

Do either the first column or the second column of question 1; then do question 2.

1. Simplify the following expressions to a single power using the Product Law.

   a. $2^5 \times 2^{11}$ b. $5^3 \times 5^{10}$

   c. $216^{32} \times 216^{16}$ d. $138^{22} \times 138^{17}$

   e. $b^7 \times b^{21}$ f. $y^{10} \times y^{12}$

   g. $m^{105} \times m^{236}$ h. $n^{352} \times n^{102}$

   i. $5^8 \times 5^{12} \times 5^3$ j. $6^{12} \times 6^7 \times 6^3$

   k. $(ab)^3 \times (ab)^{12}$ l. $\left(mn^2\right)^3 \times \left(mn^2\right)^{11}$

2. Explain what conditions must exist before you can use the Product Law.

For solutions to **Activity 1**, turn to **Appendix A, Topic 3**.

Activity 2

Use the Power of a Power Law for exponents.

Is there a short way of simplifying

$(3^2)^4$ or $(b^3)^5$?

Study these two cases to see if you can find a short way to simplify expressions like these.

Case 1

$$\begin{aligned}(3^2)^4 &= 3^2 \times 3^2 \times 3^2 \times 3^2\\ &= (3\times3)\times(3\times3)\times(3\times3)\times(3\times3)\\ &= 3^8\end{aligned}$$

$$\begin{aligned}(3^2)^4 &= 3^{2\times4}\\ &= 3^8\end{aligned}$$

It seems that the exponent of the solution is the product of the two exponents in the original expression. Now see if this is true for the second case.

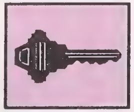
Case 2

$$\begin{aligned}(b^3)^5 &= b^3 \times b^3 \times b^3 \times b^3 \times b^3\\ &= (b\times b\times b)\times(b\times b\times b)\\ &\quad \times(b\times b\times b)\times(b\times b\times b)\times(b\times b\times b)\\ &= b^{15}\end{aligned}$$

$$\begin{aligned}(b^3)^5 &= b^{3\times5}\\ &= b^{15}\end{aligned}$$

This case has the same result.

Actually, no matter what number you choose for the exponents, you get the same result if you used the expanded form or if you multiplied the two existing exponents.



This is called the **Power of a Power Law**. It can be written as follows in general terms.

$\left(x^m\right)^n = x^{m \times n}$ where m, $n \in W$

Now add this information to the chart you started earlier.

Expanding	Number Bases	Variable Bases	Exponent Law
$2^3 \times 2^4 = 2\times2\times2\times2\times2\times2\times2$ $= 2^7$	$2^3 \times 2^4 = 2^{3+4}$ $= 2^7$	$x^3 \times x^4 = x^{3+4}$ $= x^7$	$x^m \times x^n = x^{m+n}$ Product Law (Multiplication Law)
$\left(6^2\right)^3 = \left(6^2\right)\left(6^2\right)\left(6^2\right)$ $= (6\times6)(6\times6)(6\times6)$ $= 6^6$	$\left(6^2\right)^3 = 6^{2\times3}$ $= 6^6$	$\left(x^2\right)^3 = x^{2\times3}$ $= x^6$	$\left(x^m\right)^n = x^{m\times n}$ $= x^{mn}$ Power of a Power Law

Now try some practice questions.

Do the first or second column of question 1; then do questions 2 and 3.

1. Simplify the following expressions using the Power of a Power Law.

   a. $\left(2^{3}\right)^{9}$ b. $\left(3^{5}\right)^{6}$

   c. $\left(125^{5}\right)^{30}$ d. $\left(64^{15}\right)^{25}$

   e. $\left(x^{15}\right)^{6}$ f. $\left(y^{22}\right)^{8}$

   g. $\left(a^{125}\right)^{32}$ h. $\left(m^{32}\right)^{246}$

   i. $\left[\left(5^{2}\right)^{4}\right]^{7}$ j. $\left[\left(3^{3}\right)^{4}\right]^{10}$

   k. $\left[(mn)^{2}\right]^{13}$ l. $\left[(xy)^{6}\right]^{7}$

2. Simplify the following expressions.

   a. $\left(5^{4}\right)^{2}\times\left(5^{2}\right)^{6}$

   b. $\left(3^{5}\right)^{4}\times\left(3^{6}\right)^{5}$

   c. $\left(m^{2}\right)^{6}\times\left(m^{3}\right)^{12}$

   d. $\left(a^{3}\right)^{6}\times\left(a^{5}\right)^{5}$

3. Explain how the Power of a Power Law is used.

For solutions to **Activity 2**, turn to **Appendix A, Topic 3**.

Power of a Power Law:

$\left(x^{m}\right)^{n}=x^{m\times n}$ where $m,\ n\in W$

$m,\ n\in W$ means that m and n are whole numbers.

Hint: When more than one set of brackets is involved, work from the inside brackets out when you are simplifying the expression.

Activity 3

Use the Division or Quotient Law of exponents for powers with literal bases and whole number exponents.

If the Multiplication or Product Law allows you to add exponents, will a similar law for division allow you to subtract exponents?

Look at some cases to see if this educated guess is correct.

Case 1

$$2^{10} \div 2^{6} = \frac{2^{10}}{2^{6}}$$

$$= \frac{2 \times 2 \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times 2 \times 2}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}}}$$

$$= 2 \times 2 \times 2 \times 2$$

$$= 2^{4}$$

$$2^{10} \div 2^{6} = 2^{10-6}$$

$$= 2^{4}$$

The educated guess holds true as long as the exponent of the denominator is subtracted from the exponent of the numerator and the bases are the same.

Now see if this is true for another case.

$$\underset{\text{dividend}}{\underset{\uparrow}{2^{10}}} \div \underset{\text{divisor}}{\underset{\uparrow}{2^{6}}} = \frac{2^{10} \rightarrow \text{numerator}}{2^{6} \rightarrow \text{denominator}}$$

Case 2

$$m^{13} \div m^{5} = \frac{m^{13}}{m^{5}}$$

$$= \frac{m \times m \times m \times m \times \cancelto{1}{m} \times \cancelto{1}{m} \times \cancelto{1}{m} \times \cancelto{1}{m} \times \cancelto{1}{m} \times m \times m \times m \times m}{\underset{1}{\cancel{m}} \times \underset{1}{\cancel{m}} \times \underset{1}{\cancel{m}} \times \underset{1}{\cancel{m}} \times \underset{1}{\cancel{m}}}$$

$$= m \times m \times m \times m \times m \times m \times m \times m$$

$$= m^{8}$$

$$m^{13} \div m^{5} = m^{13-5}$$

$$= m^{8}$$

This case follows the rule, as well.

This is called the **Division Law** or **Quotient Law**. It can be written as follows in general terms.

$$\frac{x^{m}}{x^{n}} = x^{m-n} \text{ or } x^{m} \div x^{n} = x^{m-n} \text{ where } x \neq 0 \text{ and } m,\ n \in W$$

Enter this rule on the chart you started earlier.

$x \neq 0$ because division by zero is undefined.

Expanding	Number Bases	Variable Bases	Exponent Law
$2^3 \times 2^4 = 2\times2\times2\times2\times2\times2\times2$ $= 2^7$	$2^3 \times 2^4 = 2^{3+4}$ $= 2^7$	$x^3 \times x^4 = x^{3+4}$ $= x^7$	$x^m \times x^n = x^{m+n}$ Product Law (Multiplication Law)
$(6^2)^3 = (6^2)(6^2)(6^2)$ $= (6\times6)(6\times6)(6\times6)$ $= 6^6$	$(6^2)^3 = 6^{2\times3}$ $= 6^6$	$(x^2)^3 = x^{2\times3}$ $= x^6$	$(x^m)^n = x^{m\times n}$ $= x^{mn}$ Power of a Power Law
$3^5 \div 3^2 = \dfrac{3\times3\times\overset{1}{\cancel{3}}\times\overset{1}{\cancel{3}}\times3}{\underset{1}{\cancel{3}}\times\underset{1}{\cancel{3}}}$ $= 3\times3\times3$ $= 3^3$	$3^5 \div 3^2 = 3^{5-2}$ $= 3^3$	$x^5 \div x^2 = x^{5-2}$ $= x^3$	$x^m \div x^n = x^{m-n}$ Quotient Law (Division Law)

Now try some questions using these laws.

Do the first or second column of questions 1 and 2; then do question 3.

1. Simplify the following expressions using the quotient law.

a. $3^5 \div 3$	b. $4^9 \div 4^0$
c. $16^{482} \div 16^{107}$	d. $23^{332} \div 23^{97}$
e. $m^{23} \div m^9$	f. $n^{35} \div n^{15}$
g. $x^{201} \div x^{157}$	h. $y^{97} \div y^{63}$
i. $3^{10} \div 3^4 \div 3^3$	j. $16^{15} \div 16^{10} \div 16^3$
k. $m^{23} \div \left(m^{24} \div m^{12}\right)$	l. $a^{17} \div \left(a^{20} \div a^{17}\right)$

2. Simplify the following expressions.

a. $\left(3^4\right)^4 \div \left(3^2\right)^3$	b. $\left(5^4\right)^3 \div \left(5^2\right)^5$
c. $16^4 \times 16^9 \div \left(16^2\right)^5$	d. $32^5 \times \left(32^{11}\right)^2 \div 32^{14}$
e. $\left(m^4 \times m^3\right)^2 \div m^{12}$	f. $m^{22} \times \left(m^4 \div m\right)^3$

3. Explain what circumstances must exist before you can use the Quotient Law.

Recall: $3 = 3^1$ exponent is 1

Hint: Be careful to use the correct operation. When dividing, the exponents are subtracted.

Quotient Law: $x^m \div x^n = x^{m-n}$

For solutions to **Activity 3**, turn to **Appendix A**, **Topic 3**.

Activity 4

Use the Power of a Product Law for exponents.

Try to develop a procedure for simplifying expressions like $(2m)^4$ and $(xy)^5$.

Look at the first case.

Case 1

$$\begin{aligned}(2m)^4 &= 2m\times 2m\times 2m\times 2m\\ &= 2\times 2\times 2\times 2\times m\times m\times m\times m\\ &= 2^4 m^4\end{aligned}$$

It appears that each factor in parentheses is raised to the same exponent that the entire product is raised to.

See if this is true in the second case, too.

Case 2

$$\begin{aligned}(xy)^5 &= xy\times xy\times xy\times xy\times xy\\ &= x\times x\times x\times x\times x\times y\times y\times y\times y\times y\\ &= x^5 y^5\end{aligned}$$

The same results were reached in this case.

$2^4 m^4$ can be simplified further to get $16m^4$.

This is called the **Power of a Product Law**. In general terms it states that $(xy)^m = x^m y^m$ where $m \in W$.

Now add this new rule to the chart you developed earlier.

Expanding	Number Bases	Variable Bases	Exponent Law
$2^3 \times 2^4 = 2\times2\times2\times2\times2\times2\times2$ $= 2^7$	$2^3 \times 2^4 = 2^{3+4}$ $= 2^7$	$x^3 \times x^4 = x^{3+4}$ $= x^7$	$x^m \times x^n = x^{m+n}$ Product Law (Multiplication Law)
$(6^2)^3 = (6^2)(6^2)(6^2)$ $= (6\times6)(6\times6)(6\times6)$ $= 6^6$	$(6^2)^3 = 6^{2\times3}$ $= 6^6$	$(x^2)^3 = x^{2\times3}$ $= x^6$	$(x^m)^n = x^{m\times n}$ $= x^{mn}$ Power of a Power Law
$3^5 \div 3^2 = \frac{3\times3\times\overset{1}{\cancel{3}}\times\overset{1}{\cancel{3}}\times3}{\underset{1}{\cancel{3}}\times\underset{1}{\cancel{3}}}$ $= 3\times3\times3$ $= 3^3$	$3^5 \div 3^2 = 3^{5-2}$ $= 3^3$	$x^5 \div x^2 = x^{5-2}$ $= x^3$	$x^m \div x^n = x^{m-n}$ Quotient Law (Division Law)
$(4\times5)^3 = (4\times5)(4\times5)(4\times5)$ $= (4\times4\times4)(5\times5\times5)$ $= 4^3 \times 5^3$	$(4\times5)^3 = 4^3 \times 5^3$	$(xy)^3 = x^3 y^3$	$(xy)^m = x^m y^m$ Power of a Product Law

Do the first or second column of questions 1 and 2; then do question 3.

1. Simplify the following expressions using the Power of a Product Law.

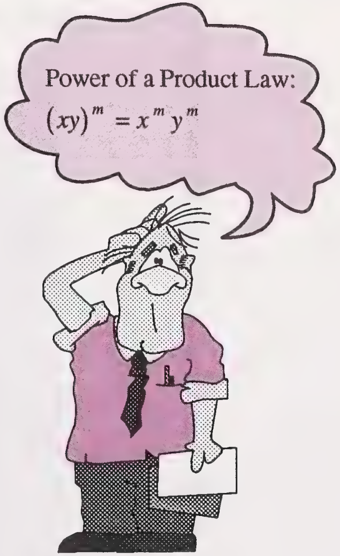
a. $(3a)^{11}$	b. $(11b)^{9}$
c. $(126x)^{503}$	d. $(302y)^{126}$
e. $(mn)^{102}$	f. $(rs)^{1026}$
g. $(14ab)^{32}$	h. $(15mn)^{16}$
i. $(abc)^{14}$	j. $(xyz)^{22}$
k. $(126rstu)^{20}$	l. $(196mnxyz)^{12}$

2. Simplify the following expressions.

a. $\left(2m^{2}\right)^{4}$	b. $\left(xy^{3}\right)^{10}$
c. $\left(2^{3}\times2^{5}m^{3}\right)^{5}$	d. $\left(5^{10}m^{4}\div5^{8}m^{2}\right)^{3}$
e. $\left(xy^{2}\right)^{3}\left(x^{3}y\right)^{4}$	f. $\left(10^{4}n^{3}\right)^{4}\div\left(10^{2}n^{2}\right)^{3}$

3. Explain what circumstances must exist before you can use the Power of a Product Law.

For solutions to **Activity 4**, turn to **Appendix A, Topic 3**.



Activity 5

Use the Power of a Quotient Law for exponents.

Can the same rule be used for moving an exponent to both the numerator and the denominator when a whole fraction is raised to the exponent?

Examine the following two cases to see what happens.

Case 1

$$\left(\frac{7}{8}\right)^5 = \frac{7}{8}\times\frac{7}{8}\times\frac{7}{8}\times\frac{7}{8}\times\frac{7}{8}$$
$$= \frac{7\times7\times7\times7\times7}{8\times8\times8\times8\times8}$$
$$= \frac{7^5}{8^5}$$

Case 2

$$\left(\frac{x}{y}\right)^7 = \frac{x}{y}\times\frac{x}{y}\times\frac{x}{y}\times\frac{x}{y}\times\frac{x}{y}\times\frac{x}{y}\times\frac{x}{y}$$
$$= \frac{x\times x\times x\times x\times x\times x\times x}{y\times y\times y\times y\times y\times y\times y}$$
$$= \frac{x^7}{y^7}$$

The educated guess holds true for both of these cases.

This rule is referred to as the Power of a Quotient Law. It is written using the following general terms

$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ where $y \neq 0$ and $m \in W$.

Now add this rule to the chart you started earlier.

Fractions such as $\frac{7}{8}$ and $\frac{x}{y}$ are also called quotients since $\frac{7}{8} = 7 \div 8$ and $\frac{x}{y} = x \div y$.

Expanding	Number Bases	Variable Bases	Exponent Law
$2^3 \times 2^4 = 2\times2\times2\times2\times2\times2\times2$ $= 2^7$	$2^3 \times 2^4 = 2^{3+4}$ $= 2^7$	$x^3 \times x^4 = x^{3+4}$ $= x^7$	$x^m \times x^n = x^{m+n}$ Product Law (Multiplication Law)
$(6^2)^3 = (6^2)(6^2)(6^2)$ $= (6\times6)(6\times6)(6\times6)$ $= 6^6$	$(6^2)^3 = 6^{2\times3}$ $= 6^6$	$(x^2)^3 = x^{2\times3}$ $= x^6$	$(x^m)^n = x^{m\times n}$ $= x^{mn}$ Power of a Power Law
$3^5 \div 3^2 = \dfrac{3\times3\times\cancel{3}^{1}\times\cancel{3}^{1}\times3}{\cancel{3}_{1}\times\cancel{3}_{1}}$ $= 3\times3\times3$ $= 3^3$	$3^5 \div 3^2 = 3^{5-2}$ $= 3^3$	$x^5 \div x^2 = x^{5-2}$ $= x^3$	$x^m \div x^n = x^{m-n}$ Quotient Law (Division Law)
$(4\times5)^3 = (4\times5)(4\times5)(4\times5)$ $= (4\times4\times4)(5\times5\times5)$ $= 4^3 \times 5^3$	$(4\times5)^3 = 4^3 \times 5^3$	$(xy)^3 = x^3 y^3$	$(xy)^m = x^m y^m$ Power of a Product Law
$\left(\dfrac{7}{8}\right)^3 = \dfrac{7}{8}\times\dfrac{7}{8}\times\dfrac{7}{8}$ $= \dfrac{7\times7\times7}{8\times8\times8}$ $= \dfrac{7^3}{8^3}$	$\left(\dfrac{7}{8}\right)^3 = \dfrac{7^3}{8^3}$	$\left(\dfrac{x}{y}\right)^3 = \dfrac{x^3}{y^3}$	$\left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m}$ Power of a Quotient Law

Now try the following questions.

Do the first or second column of questions 1 and 2; then do question 3.

1. Simplify the following expressions using the Power of a Quotient Law.

   a. $\left(\frac{5}{7}\right)^{11}$ b. $\left(\frac{3}{11}\right)^{9}$

   c. $\left(\frac{123}{562}\right)^{93}$ d. $\left(\frac{93}{256}\right)^{102}$

   e. $\left(\frac{5}{x}\right)^{26}$ f. $\left(\frac{4}{3}\right)^{32}$

   g. $\left(\frac{m}{n}\right)^{15}$ h. $\left(\frac{a}{b}\right)^{16}$

   i. $(5 \div q)^{11}$ j. $(16 \div m)^{21}$

   k. $\left(\frac{\frac{m}{5}}{5}\right)^{4}$ l. $\left(\frac{\frac{b}{c}}{c}\right)^{5}$

2. Simplify the following expressions.

   a. $\left(\frac{5}{m^2}\right)^{6}$ b. $\left(\frac{n^3}{5^2}\right)^{4}$

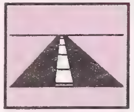
   c. $\left(\frac{2^4 m^2}{n^3}\right)^{5}$ d. $\left(\frac{3^2 a}{b^3}\right)^{4}$

   e. $\left(\frac{m^2}{n^3}\right)^{4}\left(\frac{n^2}{m^5}\right)^{3}$ f. $\left(\frac{x^3}{y}\right)^{5} \div \left(\frac{x^2}{y^4}\right)^{3}$

3. Explain how the Power of a Quotient Law is used.

For solutions to **Activity 5**, turn to **Appendix A**, **Topic 3**.

Activity 6



Verify each of the power laws for exponents.

Sometimes you may be asked to verify a law. To verify means to show that the operation or rule is true (or false) using examples.

Example 1

Verify the Product Law for exponents.

$x^m x^n = x^{m+n}$ where $m, n \in W$

- Show that $x^2 \times x^5 = x^7$.

Solution

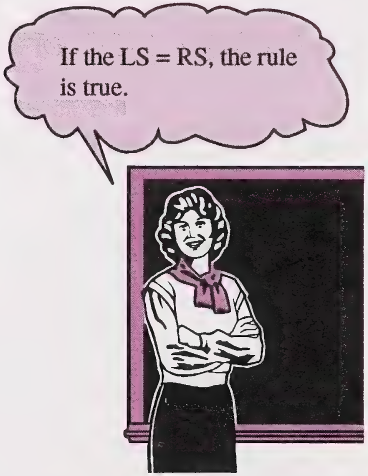
LS		RS
$x^2 \times x^5$		x^7
$(x \times x)(x \times x \times x \times x \times x)$		x^7
x^7		x^7
LS	=	RS

Since the left side is equal to the right side, the rule is true.

- Show that $8^2 \times 8^1 = 8^3$.

Solution:

LS		RS
$8^2 \times 8^1$		8^3
$(8 \times 8) \times (8)$		8^3
8^3		8^3
512		512
LS	=	RS



Example 2

Verify the Power of a Power Law for Exponents.

$\left(x^m\right)^n = x^{m \times n}$ where $m, n \in W$

- Show that $\left(x^3\right)^2 = x^6$.

Solution:

LS	RS
$\left(x^3\right)^2$	x^6
$x^3 \times x^3$	x^6
$(x \times x \times x) \times (x \times x \times x)$	x^6
x^6	x^6
LS =	RS

Since the left side is equal to the right side, the rule is true.

- Show that $\left(5^4\right)^3 = 5^{12}$

Solution:

LS	RS
$\left(5^2\right)^3$	5^6
$\cdot\, 5^2 \times 5^2 \times 5^2$	5^6
$(5 \times 5) \times (5 \times 5) \times (5 \times 5)$	5^6
5^6	5^6
LS =	RS

Since the left side is equal the right side, the rule is true.

Notice that what you are doing here is similar to what was done in each of the simplification examples in the previous activities. Each of the Power Laws for exponents can be verified in this way. Try to verify the remaining three exponent laws on your own.

Now try the questions that follow.

1. Verify each of the following.

   a. $2^3 \times 2^1 = 2^4$

   b. $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$

   c. $\left(x^3\right)^3 = x^9$

   d. $(3n)^5 = 3^5 n^5$

   e. $a^{12} \div a^4 = a^8$

   f. $4^9 \div 4^5 = 4^4$

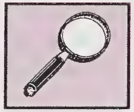
2. Verify that $\frac{a^m \bullet a^n}{a^x} = a^{(m+n-x)}$.

For solutions to **Activity 6**, turn to the **Appendix A**, **Topic 3**.

If you require help, do the Extra Help section.

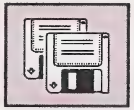
If you want more challenging explorations, do the Extensions section.

$\Big\}$ You may decide to do both.



Extra Help

Do either **Part A** or **Part B**.

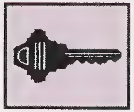


Part A

Use the computer diskette entitled *Algebra Drill and Practise II*[1] to increase your understanding of the laws of exponents. Do only section 2 called Integral Exponents from this diskette.

Part B

The ability to work with powers is essential to a good understanding of polynomials. The five power laws discussed in this topic are outlined in this section.



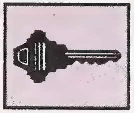
To multiply two powers, **add** the exponents. This rule only works if the **bases** are the same.

(Remember that in 2^3, 2 is called the base.)

$$5^3 \times 5^4 = 5^{3+4} \qquad a^9 \times a^4 = a^{9+4}$$
$$= 5^7 \qquad = a^{13}$$

If the bases are not the same, then the exponents cannot be added.

For example, $5^2 \times 3^4 = 5^2 \times 3^4$. (This expression cannot be simplified to a single power because the bases are different.)



To divide two powers, **subtract** the bottom exponent from the top exponent. Do not subtract the top exponent from the bottom one. Again, this rule only works when both bases are the same.

$$\frac{5^7}{5^4} = 5^{7-4} \qquad \frac{x^{10}}{x^8} = x^{10-8}$$
$$= 5^3 \qquad = x^2$$

[1] Algebra Drill and Practise II is a title of *Conduit Publishers*

To find the power of a power, **multiply** the exponents.

$$\left(2^3\right)^4 = 2^{3\times4} = 2^{12}$$

$$\left(x^2\right)^5 = x^{2\times5} = x^{10}$$

To find the power of a product, **raise** each factor to that power.

$$\left(2x^3\right)^3 = (2)^3 \times \left(x^3\right)^3 = 2^3 x^9$$

$$\left(x^2 y^3\right)^4 = \left(x^2\right)^4 \left(y^3\right)^4 = x^8 y^{12}$$

To find the power of a quotient, **raise** the numerator and denominator to that power.

$$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$$

$$\left(\frac{x^2}{y^3}\right)^4 = \frac{\left(x^2\right)^4}{\left(y^3\right)^4} = \frac{x^8}{y^{12}}$$

Now practice working with these laws in the exercises that follow.

1. Multiply the powers.

   a. $x^5 \times x^7$
   b. $a^4 \times a^{11}$
   c. $2^4 \times 2^{19}$
   d. $y^{14} \times y^{26}$

2. Divide the powers.

   a. $\dfrac{x^{14}}{x^9}$
   b. $\dfrac{y^{24}}{y^{16}}$
   c. $\dfrac{5^7}{5^3}$
   d. $\dfrac{k^{100}}{k^{25}}$
   e. $\dfrac{z^{17}}{z^{14}}$

3. Find the power of each power.

   a. $\left(2^5\right)^2$
   b. $\left(x^4\right)^7$
   c. $\left(y^3\right)^2$
   d. $\left(z^5\right)^7$
   e. $\left(k^9\right)^4$

4. Find the power of each product.

a. $(3y^2)^4$

b. $(x^3y^5)^2$

c. $(k^4z^5)^2$

d. $(2k^3z^2)^3$

5. Find the power of each quotient.

a. $\left(\frac{x}{y}\right)^2$

b. $\left(\frac{x^2}{y^3}\right)^2$

c. $\left(\frac{5^3}{3^4}\right)^2$

d. $\left(\frac{x^3}{k^2}\right)^4$

For solutions to **Extra Help**, turn to **Appendix A, Topic 3**.

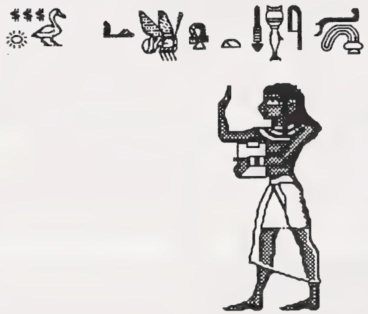
Extensions

After having saved the life of the Pharoah of Egypt, Jacob Aesop was offered two choices as a reward.

- He could accept one million dollars in cash as a reward.
- He could receive one cent on the first day of August, two cents on the second day, four cents on the next day, eight cents on the next and so forth until the end of August. Each day the amount of money would double from the previous day.

If Jacob was a knowledgable mathematician and interested in monetary gains, which alternative should he choose?

For solutions to **Extensions**, turn to **Appendix A, Topic 3**.



Topic 4 Zero and Negative Exponents

Introduction

Here is an extension of some of the previous work on exponents. These skills are used in furture math courses, as well as in many areas of science and industry.

What Lies Ahead

Throughout this topic you will learn to

1. use zero exponents

2. use negative exponents

Now that you know what to expect, turn the page to begin your study of zero and negative exponents.

Exploring Topic 4

Activity 1

Use zero exponents.

What do you think 2^0 equals? The following step by step pattern may help you find the answer.

$$2^4 = 16$$
$$2^3 = 8$$
$$2^2 = 4$$
$$2^1 = 2$$
$$2^0 = ?$$

What do you think the next value should be?

In the second column, each value is one-half of the previous number. Therefore, the last value should be one-half of 2, which is 1. This same solution can be demonstrated in another way.

You know from a previous definition that a number divided by itself is always equal to one. By using the laws of exponents, examine the following case that demonstrates 2^0 is equal to one.

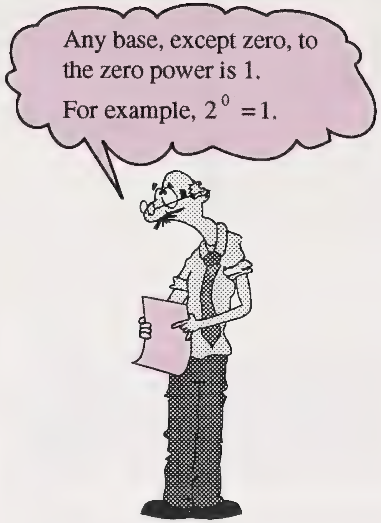
$$\begin{aligned} 1 &= \frac{2}{2} \\ &= \frac{2^1}{2^1} \\ &= 2^{1-1} \\ &= 2^0 \end{aligned}$$

This same situation can be demonstrated for all numbers and variables, except when zero is the base.

This can be shown in general terms.

$x^0 = 1$ where $x \neq 0$

Take a look at some examples using this rule.



Example 1

Evaluate the following expressions.

- 5^0

  Solution:

  $5^0 = 1$

- 26^0

  Solution:

  $26^0 = 1$

- y^0

  Solution:

  $y^0 = 1$

- $(mn)^0$

  Solution:

  $$\begin{aligned}(mn)^0 &= m^0 n^0\\ &= 1\times 1\\ &= 1\end{aligned}$$

Now try the practice exercises.

Do either the first or second column of question 1; then do question 2.

1. Simplify the following expressions.

a. 5^0	b. 12^0
c. 256^0	d. 192^0
e. x^0	f. d^0
g. $(2m)^0$	h. $(xy)^0$
i. $\left(\frac{16x}{y}\right)^0$	j. $\left(\frac{m^2}{7n}\right)^0$
k. $2m^0$	l. $\frac{x^0}{y}$
m $\left[\left(2m^2\right)^0\right]^2$	n. $2m^0 \times 15nm^2$

2. Explain why you cannot permit the base to be zero with an exponent of zero.

For solutions to **Activity 1**, turn to **Appendix A, Topic 4**.

Activity 2

Use negative exponents.

Now extend the pattern you used in the last section. The exponent for the base will continue to be decreased by one and the evaluated number will be halved for each step.

2^4	16
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2}$ or 0.5
2^{-2}	$\frac{1}{4}$ or 0.25
2^{-3}	$\frac{1}{8}$ or 0.125
2^{-4}	$\frac{1}{16}$ or 0.0625

Did you notice that 2^{-1} is the reciprocal of 2^1, and 2^{-2} is the reciprocal of 2^2?

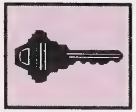
This pattern can be demonstrated using the exponent laws. Examine the following two cases.

Case 1

$$\begin{aligned} 2^{-2} &= 2^{2-4} \\ &= \frac{2^2}{2^4} \\ &= \frac{4}{16} \\ &= \frac{1}{4} \\ &= \frac{1}{2^2} \end{aligned}$$

Case 2

$$\begin{aligned} 2^{-4} &= 2^{2-6} \\ &= \frac{2^2}{2^6} \\ &= \frac{4}{64} \\ &= \frac{1}{16} \\ &= \frac{1}{2^4} \end{aligned}$$



In general, the negative exponent means to take the reciprocal of the number. It is written in general terms as $x^{-m} = \frac{1}{x^m}$ where $x \neq 0$, $m \in I$.

Now look at some more examples.

Remember: When you multiply a number by its reciprocal, the answer is 1.

$$\left.\begin{aligned} 2^{-1} &= \frac{1}{2} \\ 2^1 &= 2 \end{aligned}\right\} \text{ These are reciprocals since } \frac{1}{2} \times 2 = 1.$$

$$\left.\begin{aligned} 2^{-2} &= \frac{1}{4} \\ 2^2 &= 4 \end{aligned}\right\} \text{ These are reciprocals since } \frac{1}{4} \times 4 = 1.$$

Example 2

- Evaluate or simplify 3^{-3}.

Solution:

$$3^{-3} = \frac{1}{3^3}$$
$$= \frac{1}{27}$$

- Evaluate or simplify $2^{-3}4^2$.

Solution:

$$2^{-3}4^2 = \frac{4^2}{2^3}$$
$$= \frac{16}{8}$$
$$= 2$$

- Evaluate or simplify x^{-3}.

Solution:

$$x^{-3} = \frac{1}{x^3}$$

- Evaluate or simplify $\frac{3a^{-2}}{b^{-3}}$.

Solution:

$$\frac{3a^{-2}}{b^{-3}} = \frac{3b^3}{a^2}$$

- Evaluate or simplify $\left(a^{-2}b^{-1}\right)^{-3}\left(a^3b^2\right)^{-2}$.

Solution:

$$\left(a^{-2}b^{-1}\right)^{-3}\left(a^3b^2\right)^{-2} = \left(a^{-2}\right)^{-3}\left(b^{-1}\right)^{-3}\left(a^3\right)^{-2}\left(b^2\right)^{-2}$$
$$= a^6b^3a^{-6}b^{-4}$$
$$= a^{6+(-6)}b^{3+(-4)}$$
$$= a^0b^{-1}$$
$$= 1 \times b^{-1}$$
$$= b^{-1}$$
$$= \frac{1}{b}$$

Now add these two new rules to your chart.

Expanding	Number Bases	Variable Bases	Exponent Law
$2^3 \times 2^4 = 2\times2\times2\times2\times2\times2\times2$ $= 2^7$	$2^3 \times 2^4 = 2^{3+4}$ $= 2^7$	$x^3 \times x^4 = x^{3+4}$ $= x^7$	$x^m \times x^n = x^{m+n}$ Product Law (Multiplication Law)
$(6^2)^3 = (6^2)(6^2)(6^2)$ $= (6\times6)(6\times6)(6\times6)$ $= 6^6$	$(6^2)^3 = 6^{2\times3}$ $= 6^6$	$(x^2)^3 = x^{2\times3}$ $= x^6$	$(x^m)^n = x^{m\times n}$ $= x^{mn}$ Power of a Power Law
$3^5 \div 3^2 = \frac{3\times3\times\cancel{3}^1\times\cancel{3}^1\times3}{\cancel{3}_1\times\cancel{3}_1}$ $= 3\times3\times3$ $= 3^3$	$3^5 \div 3^2 = 3^{5-2}$ $= 3^3$	$x^5 \div x^2 = x^{5-2}$ $= x^3$	$x^m \div x^n = x^{m-n}$ Quotient Law (Division Law)
$(4\times5)^3 = (4\times5)(4\times5)(4\times5)$ $= (4\times4\times4)(5\times5\times5)$ $= 4^3\times5^3$	$(4\times5)^3 = 4^3\times5^3$	$(xy)^3 = x^3y^3$	$(xy)^m = x^my^m$ Power of a Product Law
$\left(\frac{7}{8}\right)^3 = \frac{7}{8}\times\frac{7}{8}\times\frac{7}{8}$ $= \frac{7\times7\times7}{8\times8\times8}$ $= \frac{7^3}{8^3}$	$\left(\frac{7}{8}\right)^3 = \frac{7^3}{8^3}$	$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$	$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ Power of a Quotient Law
	$2^0 = 1$	$x^0 = 1$ When $x \neq 0$	$x^0 = 1$ Zero Exponent
	$2^{-3} = \frac{1}{2^3}$	$x^{-3} = \frac{1}{x^3}$ When $x \neq 0$	$x^{-m} = \frac{1}{x^m}$ Negative Exponent

Now try the questions that follow.

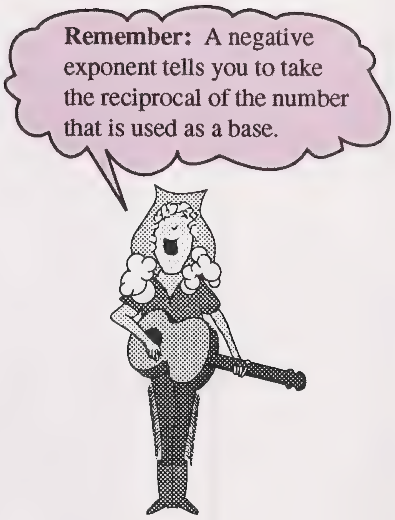
Do either the first or second column of question 1; then do question 2.

1. Simplify the following expressions to have only positive exponents.

a. 2^{-8}	b. 15^{-20}
c. 293^{-92}	d. 302^{-1024}
e. a^{-13}	f. m^{-26}
g. g^{-249}	h. n^{-1029}
i. $(2m)^{-14}$	j. $\left(\frac{5}{y}\right)^{-3}$
k. $\left(x^{-3}y^{2}\right)^{3}\left(x^{2}y^{-3}\right)^{2}$	l. $\dfrac{a^{-2}b}{a^{-4}b^{-3}}$
m. $\left(2m^{2}n^{2}\right)^{-1} \div \left(2m^{2}y\right)^{-2}$	n. $\left(\dfrac{x^{-2}y}{z^{3}}\right)^{-3} \div \left(\dfrac{x^{5}y^{-2}}{z^{3}}\right)^{-1}$

2. Explain the difference between a negative exponent and a negative base.

For solutions to **Activity 2**, turn to **Appendix A, Topic 4**.



If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

Many students have difficulty in evaluating powers with negative bases or negative exponents. Your scientific calculator has the ability to do these evaluations and should be used whenever possible. The following examples show how to evaluate powers that have negative bases or negative exponents.

The $\boxed{x^y}$ key is used to evaluate powers.

The $\boxed{+/-}$ key changes a positve number to its negative and vice versa.

Example 3

Evaluate $(-2)^5$.

Solution:

Enter	Display
2	2
$\boxed{+/-}$	-2
$\boxed{x^y}$	-2
5	5
$\boxed{=}$	-32

Example 4

Evaluate $(-2)^{-3}$.

Solution:

Enter	Display
2	2
+/−	−2
x^y	−2
3	3
+/−	−3
=	−0.125

Example 5

Evaluate $\frac{1}{(3)^{-5}}$.

Solution:

Enter	Display
1	1
÷	1
3	3
x^y	3
5	5
+/−	−5
=	243

Example 6

Evaluate $\frac{1}{(-2)^3} \times (-3)^2$.

Solution:

Enter	Display
1	1
$\boxed{\div}$	1
2	2
$\boxed{+/-}$	-2
$\boxed{x^y}$	-2
3	3
$\boxed{\times}$	-0.125
3	3
$\boxed{+/-}$	-3
$\boxed{x^y}$	-3
2	2
$\boxed{=}$	-1.125

Note: If your calculator displays an error message (E) when you try to calculate $\frac{1}{(-2)^3}$, press the $\boxed{=}$ key after you divide 1 by 2, as shown.

Enter	Display
1	1
$\boxed{\div}$	1
2	2
$\boxed{=}$	0.5
$\boxed{+/-}$	-0.5
$\boxed{x^y}$	-0.5
3	3
$\boxed{\times}$	-0.125
3	3
$\boxed{+/-}$	-3
$\boxed{x^y}$	-3
2	2
$\boxed{=}$	-1.125

Do the following questions. Use a calculator to evaluate each power.

1. 5^{-2}
2. $(-5)^{2}$
3. $(-5)^{-2}$
4. $\frac{1}{(2)^{3}}$
5. $\frac{1}{(-2)^{3}}$
6. $\frac{1}{(-2)^{-3}}$

For solutions to **Extra Help**, turn to **Appendix A, Topic 4**.

Extensions

Evaluating Powers with Negative Numbers

If you already know how to evaluate powers with negative numbers using a scientific calculator, then do the exercises that follow. If you do not know how, then go over the preceding **Extra Help** section before you attempt the exercises.

Evaluate the following powers. Use a calculator and round your answer to one decimal place.

1. $(1.7)^{-3} \times (4.2)^{2}$
2. $(2.4)^{-1.7} \times (4.3)^{2.6}$
3. $\frac{2.4}{(5.2)^{-0.9}} \times (1.5)^{-3}$
4. $(1.7)^{2.3} \times (2.4)^{3.1} \times (1.1)^{-7.1}$

For solutions to **Extensions**, turn to **Appendix A, Topic 4**.

Scientific Notation

When dealing with the real world, you are often required to write very large and very small numbers. Here are a few examples of the types of numbers that you are forced to deal with.

The mass of the earth is 5 980 000 000 000 000 000 000 000 kg.

The mass of an electron is
0.000 000 000 000 000 000 000 000 000 000 911 kg.

Is there a shorter way to write down these number?

It would be unreasonable to expect you to continually write down numbers with so many digits. For this reason a shorthand method was developed to write this type of number.

Notice that both of these numbers contain a large number of zeros. The zero digits will be removed from the numbers by using scientific notation.

5 980 000 000 000 000 000 000 000 can be written as the product of $5.98 \times 1\,000\,000\,000\,000\,000\,000\,000\,000$.

1 000 000 000 000 000 000 000 000 can be written as 10^{24}. Now 5 980 000 000 000 000 000 000 000 can be written as the product of 5.98×10^{24}.

The earth has a mass of 5.98×10^{24} kg.

This shorthand for writing numbers is called **scientific notation.**

What characteristics distinguish scientific notation from other forms of representing numbers?

Numbers that are written in scientific notation have some common characteristics.

- The first factor is always written as a number between 1 and 10.
- The second factor is always written as a power of 10.

This is still a long process to find the scientific notation form of a number. Is there a shorter way of finding the scientific notation form?

The following short cut can be used to find the scientific notation form of a number.

Step 1: Write the first factor of the number with a decimal point after the first digit and discard any superflous zeros.

Step 2: Count the number of digits the decimal point had to move to get to this point.

Step 3: Write down this number as the exponent to the base 10 in the second factor.

Step 4: Leave the exponent positive if you moved the decimal point to the left. Make the exponent negative if you moved the decimal point to the right. (You may want to remember that a very large number has a positive exponent and a very small number has a negative exponent.)

Now change the number that represent the mass of the electron into scientific notation using this process.

0.000 000 000 000 000 000 000 000 000 000 911 kg

Step 1: The first factor is 9.11. (a number between 1 and 10.

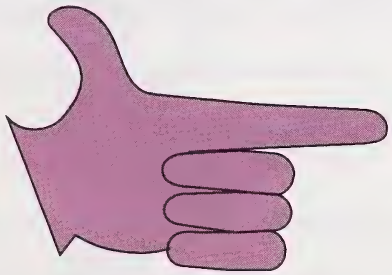
Step 2: The decimal point had to be moved 31 places to the right.

Step 3: The second factor will have an exponent of 31.

Step 4: The decimal point was moved to the right, so the exponent is negative. In scientific notation, the number is 9.11×10^{-31}.

How is a number changed back into standard form?

To change a number expressed in scientific notation, write it in expanded form by moving the decimal point the appropriate number of places to the right or to the left.



Example 7

- Write 4.56×10^{12} in expanded form.

Solution:

$$\begin{aligned} 4.56 \times 10^{12} &= 4.56 \times 1\,000\,000\,000\,000 \\ &= 4\,560\,000\,000\,000 \end{aligned}$$

- Write 1.046×10^{-5} in expanded form.

Solution:

$$\begin{aligned} 1.046 \times 10^{-5} &= 1.046 \times 0.000\,01 \\ &= 0.000\,010\,46 \end{aligned}$$

How do you multiply numbers that are in scientific notation?

Take a look at a situation and see if there is a pattern.

$$\begin{aligned} \left(3.2 \times 10^{5}\right) \times \left(2.4 \times 10^{4}\right) &= 320\,000 \times 24\,000 \\ &= 7\,680\,000\,000 \\ &= 7.68 \times 10^{9} \end{aligned}$$

Note that $3.2 \times 2.4 = 7.68$ and $10^{5} \times 10^{4} = 10^{9}$.

This situation indicates that the product of the first numbers is taken and the exponents to the base ten are added.

$$\left(2\times10^{3}\right)\times\left(4\times10^{5}\right)=(2\times4)\times10^{3+5}$$

Examine the following example to see if this procedure will continue to work.

Example 8

- Find the product of $\left(4.35\times10^{9}\right)\times\left(6.81\times10^{8}\right)$.

Solution:

$$\begin{aligned}\left(4.35\times10^{9}\right)\times\left(6.81\times10^{8}\right)&=(4.35\times6.81)\times\left(10^{9}\times10^{8}\right)\\&=29.6235\times10^{9+8}\\&=29.6235\times10^{17}&&\text{(The first number is not between 1 and 10.}\\&=\left(2.962\,35\times10^{1}\right)\times10^{17}&&\text{This number must be changed into scientific notation.)}\\&=2.962\,35\times10^{1+17}&&\text{(Add the exponents.)}\\&\doteq2.96\times10^{18}&&\text{(rounded to three significant digits)}\end{aligned}$$

If needed, see the **Extensions** section entitled Significant Digits on how to round to the correct number of significant digits.

- Find the product of $7\,650\,000 \times 0.000\,000\,0512$. Round your answer to three significant digits.

Solution:

$$\begin{aligned} 7\,650\,000 \times 0.000\,000\,0512 &= (7.65\times10^{6})\times(5.12\times10^{-8}) \\ &= (7.65\times5.12)\times(10^{6}\times10^{-8}) \\ &= (39.168)\times10^{6+(-8)} \\ &= (3.9168\times10^{1})\times10^{-2} \\ &= 3.9168\times10^{1+(-2)} \\ &= 3.9168\times10^{-1} \\ &\doteq 0.392 \quad \text{(standard form)} \end{aligned}$$

Can a similar procedure be followed for dividing numbers in scientific notation?

Examine another example to see if a similar procedure can be used.

Example 9

- Find the quotient of $(5.12\times10^{8})\div(1.6\times10^{5})$.

Solution:

$$\begin{aligned} (5.12\times10^{8})\div(1.6\times10^{5}) &= 512\,000\,000 \div 160\,000 \\ &= 3200 \\ &= 3.2\times10^{3} \end{aligned}$$

Note that $5.12\div1.6=3.2$ and $10^{8}\div10^{5}=10^{3}$.

The same procedure holds true here. When you divide, you can subtract the exponents.

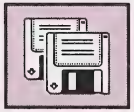
Example 10

Find the quotient of $0.000\,000\,141 \div 940\,000$.

Solution:

$$\begin{aligned} 0.000\,000\,141\div940\,000 &= (1.41\times10^{-7})\div(9.4\times10^{5}) \\ &= \frac{1.41}{9.4}\times\frac{10^{-7}}{10^{5}} \quad \text{(Write the quotient in } \tfrac{a}{b} \text{ form.)} \\ &= 0.15\times10^{-7-5} \\ &= 0.15\times10^{-12} \quad \text{(not in scientific notation)} \\ &= (1.5\times10^{-1})\times(10^{-12}) \\ &= 1.5\times10^{-1+(-12)} \\ &= 1.51\times10^{-13} \end{aligned}$$

If you would like to see how to use scientific notation on a calculator, go to the **Extensions** section entitled using Scientific Notation on your calculator.



For additional help on scientific notation and multiplication and division of numbers written in scientific notation, you may refer to the computer diskette titled *Exponential Notation*[1].

[1] Exponential Notation is a title of *Merlin Scientific Ltd.*

Now try some questions that will let you use this knowledge.

Do any three from each question.

5. Express the following numbers using scientific notation.

   a. The radius of the earth is 6 370 000 m.

   b. An electron volt is 0.000 000 000 000 000 000 160 J.

   c. The speed of light is 300 000 000 m/s.

   d. The acceleration due to gravity is 9.81 m/s^2.

   e. One astronomical unit is 300 000 000 000 m.

6. Express the following numbers using expanded form.

   a. The mass of a jumbo jet is 3.56×10^5 kg.

   b. The mass of a proton is 1.67×10^{-27} kg.

   c. Rydberg's constant is 1.1×10^7/m.

   d. The specific gravity of water is 1.00×10^0.

7. Simplify the following. Write the solutions using scientific notation.

   a. $590\,000 \times 120\,400$

   b. $0.000\,001\,03 \times 0.000\,34$

   c. $0.000\,102 \times 3\,000\,000$

   d. $340\,000 \div 0.000\,068$

   e. $0.000\,098 \div 24\,500\,000$

8. Simplify the following. Leave the solutions in scienfitic notation. Round the first factor of your solutions to one decimal place.

   a. $\dfrac{275\,000 \times 67\,000}{285\,000}$

   b. $\dfrac{189\,000 \times 0.004}{149\,000}$

   c. $\dfrac{78\,500}{34\,000 \times 0.004\,82}$

   d. $\dfrac{0.000\,004\,81}{0.000\,241 \times 340\,000}$

   e. $\dfrac{1}{78\,000 \times 0.000\,0012}$

9. Simplify the following. Leave the solutions in scientific notation. Round the first factor of your solution to one decimal place.

a. $\dfrac{(7.25\times10^{12})\times(4.05\times10^{5})}{3.098\times10^{-16}}$

b. $\dfrac{(3.4\times10^{-3})\times(9.00\times10^{-12})}{4.009\times10^{-8}}$

c. $\dfrac{(1.0\times10^{4})\times(3\times10^{34})}{2.3\times10^{20}}$

d. $\dfrac{(2.3\times10^{109})\times(4.5\times10^{209})}{4.098\times10^{99}}$

10. Solve the problems and express your solutions in scientific notation.

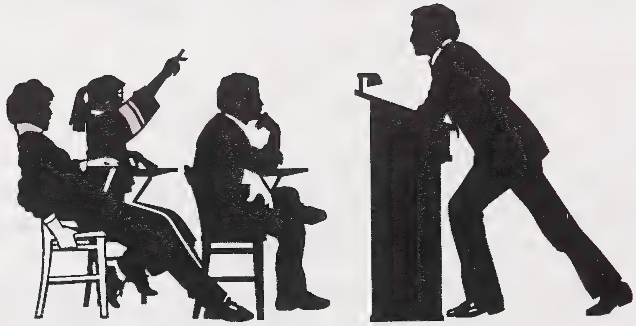
a. If you spend six hours a day and 200 days a year in a classroom, how many seconds have you spent in a classroom after twelve years?

b. If the thickness of a piece of paper is 9.0×10^{-3} cm, how many pages are in a book 7 cm thick? Round the first factor in your solution to the nearest whole number.

c. The Andromeda galaxy is 2.2×10^{6} light years away from Earth. The speed of light is 3.0×10^{8} m/s. How far in metres will light travel from the Androma galaxy to Earth? Round the first factor to one decimal place.

d. If 6.02×10^{23} atoms of oxygen have a mass of 16 g, what is the mass of one atom? Round the first factor to one decimal place.

For solutions to **Extensions**, turn to **Appendix A, Topic 4**.



Using Scientific Notation on Your Calculator

Calculators that are capable of storing very large and very small numbers use a particular screen to display numbers in scientific form. Try doing this multiplication on your calculator to see what your display looks like.

$8\,000\,000 \times 4\,000\,000 =$

Your calculator should display the numbers 3.2 and 13. The 3.2 is the first factor and the 13 is the exponent of the base 10. This display will represent the number 3.2×10^{13}.

The following procedure is usually used to enter a number written in scientific notation into a calculator.

- Enter the first factor.
- Press the exponent key.
- Enter the exponent of the second factor.

At this point, carry on with the operation you were going to use. (If this fails to work on your calculator, read your instruction manual.)

Example 11

Find the solutions to the followng using your calculator. Round the first factor of your answer to two decimal places.

- $\left(4.9 \times 10^{7}\right) \times \left(3.2 \times 10^{14}\right)$

Solution:

Enter	Display
4.9	4.9
$\boxed{\text{EXP}}$	4.9 00
7	4.9 07
$\boxed{\times}$	49 000 000
3.2	3.2
$\boxed{\text{EXP}}$	3.2 00
14	3.2 14
$\boxed{=}$	1.568 22

$$\therefore \left(4.9 \times 10^{7}\right) \times \left(3.2 \times 10^{14}\right) \doteq 1.57 \times 10^{22}$$

Note: When you press the $\boxed{\times}$ key, some calculators will display the number in scientific notation as 4.9 07, and others will display it in standard form as 49 000 000.

- $6.7\times10^{25} \div 3.8\times10^{9}$

Solution:

Enter	Display
6.7	6.7
EXP	6.7 00
25	6.7 25
÷	6.7 25
3.8	3.8
EXP	3.8 00
9	3.8 09
=	1.7631578 16

$\therefore 6.7\times10^{25} \div 3.8\times10^{9} \doteq 1.76\times10^{16}$

Example 12

Find the solution to the following question. Round the first factor of your solution in scientific notation to one decimal place.

$$\frac{489\,000\,000\,000\times0.000\,000\,18}{326\,000}$$

Solution:

$$\frac{489\,000\,000\,000\times0.000\,000\,18}{326\,000}$$
$$=\frac{\left(4.89\times10^{11}\right)\times\left(1.8\times10^{-7}\right)}{3.26\times10^{5}}$$

Enter	Display
4.89	4.89
EXP	4.89 00
11	4.89 11
×	4.89 11
1.8	1.8
EXP	1.8 00
7	1.8 07
+/–	1.8 –07
÷	88020
3.26	3.26
EXP	3.36 00
5	3.26 05
=	0.27

Note: When you press the EXP key on your calculator, the display may vary. On some calculators you will see,

Enter	Display
6.7	6.7
EXP	6.7^{00}
25	6.7^{25}

On others you will see,

Enter	Display
6.7	6.7
EXP	6.7 00
25	6.7 25

Try the questions that follow.

Do the even or odd numbered problems from question 11, and two problems from question 12.

11. Perform the following calculations with your calculator. Round the first factor of your solution to the nearest hundredth.

 a. $9\,350\,000\,000 \times 1\,350\,000\,000$

 b. $0.000\,000\,0451 \times 0.000\,000\,000\,009\,28$

 c. $8\,550\,000\,000\,000 \div 2\,850\,000\,000$

 d. $0.000\,000\,000\,009\,513 \div 3\,171\,000\,000$

 e. $\left(7.29 \times 10^{21}\right) \times \left(4.21 \times 10^{15}\right)$

 f. $\left(3.82 \times 10^{15}\right) \times \left(4.008 \times 10^{-22}\right)$

 g. $\left(9.28 \times 10^{42}\right) \div \left(1.856 \times 10^{19}\right)$

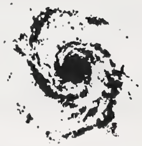
 h. $\left(1.0092 \times 10^{-18}\right) \div \left(2.0184 \times 10^{-12}\right)$

 i. $\dfrac{9\,870\,000\,000 \times 1\,025\,000\,000}{9\,085\,000\,000\,000}$

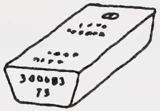
 j. $\dfrac{598\,200\,000\,000}{7\,003\,900\,000 \times 10\,940\,000\,000}$

12. Many scientific and mathematical problems involve very large or very small numbers. Do the following problems using your scientific calculator. You should use scientific notation where possible. Round your answer to the nearest tenth.

 a. The Cluster 1448 Galaxy is 2.6×10^{9} light years from Earth. If light travels at 3.0×10^{5} km/s, then how far in km is this Galaxy from Earth?



 b. The 1987 gold reserve for the Bank of Canada was 1.87×10^{7} troy ounces. If each ounce was worth \$546 Canadian dollars then, what was the worth of the 1987 gold reserve?

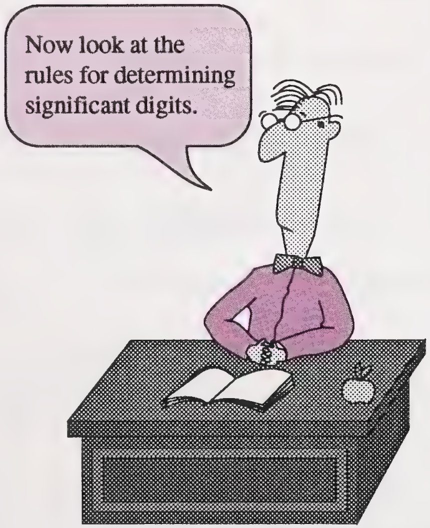


 c. In physics, the formula $v = f\lambda$ is used to show the relationship between the velocity (v), the frequency (f), and the wavelength (λ) of light rays. For gamma rays find the frequency when $v = 3 \times 10^{8}$ and $\lambda = 2.3 \times 10^{-12}$.

For solutions to **Extensions**, turn to **Appendix A, Topic 4**.

Significant Digits

Suppose you have a measurement of 2867 mm. This measurement could be expressed as 286.7 cm or 28.67 dm or 2.867 m. These measurements contain exactly the same four digits. Only the position of the decimal differs. The four digits 2, 8, 6, and 7 are called **significant digits**. Consider measurements to be **equally accurate** if they contain the same number of significant digits. The measurements previously listed are equally accurate since each has four significant digits.



Rules for Determining Significant Digits

1. Any nonzero digit is significant since it is part of the actual measurement.

2. Zero digits may or may not be significant, depending on their position in the number.

   a. Any zero digits between nonzero digits **are** significant, because they are part of the actual measurement.

   b. Zeros on the right in whole numbers **are not** significant (unless information to the contrary is given), since these zeros usually indicate that the measurement has been rounded.

   c. Zero digits on the right in decimal numbers **are** significant, since they indicate the number is accurate to that number of decimal places.

   d. Zero digits on the left in decimal numbers are not significant they only function as placeholders.

Note which zeros are significant and which are not significant in the following numbers.

1003	These zeros are significant. (See Rule 2a.)
3500	These zeros are not significant. (See Rule 2b.)
5.4870	This zero is significant. (See Rule 2c.)
402	This zero is significant. (See Rule 2a.)
0.006 27	These zeros are not significant. (See Rule 2d.)

Now try the question that follows.

13. Write **yes** if the boldface zeros are significant and **no** if they are not. State which rules apply.

Number	Significant, yes or no	Rule
1.076		
3.0704		
40.800		
0.**00**5 03		
6.**00**		
0.69		

For solutions to **Extensions**, turn to **Appendix A, Topic 4**.

There are occasions when it is difficult to determine if the zeros are significant or not. In the measurement 6600 m, you cannot immediately tell whether these zeros are significant. In such a case, always consider the zeros to be placeholders, unless some indication of the accuracy of the measurement is given. The accuracy could be indicated by an expression such as 6600 ± 0.5 m which tells you that the measurement is accurate at the 1 m level. In this case both zeros would be significant.

The best way to indicate that zeros are significant is to write the measurement in scientific notation. For example, the number 6600 could be written as 6.600×10^3. The zeros in the first factor would be considered significant according to Rule 2c.

To find how many significant digits there are in any measurement, first decide if any zeros are significant; then add the number of significant zeros to the number of nonzero digits.

14. For each measurement, circle the zeros that are not significant. Then state the number of significant digits in each measurement.

Measurement	Number of Significant Digits
a. 3.5607 g	______
b. 6.05×10^5 mL	______
c. 620.8 L	______
d. 48 052.36 g/cm^3	______
e. 0.006 048 m	______
f. 4.306 00 L	______
g. 0.346 800 cm^2	______
h. 0.000 300 kg	______
i. 6.70×10^4 mm	______
j. 4.03×10^5 t (tonne)	______
k. 6.000×10^{-2} m^3	______
l. 0.047 cm	______

For solutions to **Extensions** , turn to **Appendix A, Topic 4**.

Adding or Subtracting Measured Quantities

When adding or subtracting measured quantities, the calculated answer should be rounded to the same number of significant digits as the number with the least decimal places.

$$\begin{array}{rl} 38.5 & \text{mm} \quad \text{(least number of decimal places)} \\ 0.123 & \text{mm} \\ 19.50 & \text{mm} \\ \hline 58.123 & \text{mm} \end{array}$$

The answer should be rounded to 58.1 mm.

Multiplying and Dividing Measurements

When you multiply or divide measurements, our answer should not have more significant digits than any of the original measurements. If you leave your answer with more significant digits than any of the original measurements, it will appear more accurate than it really is.

For a given problem you should determine which measurement has the least number of significant digits, and then round the final answer to this same number of significant digits. Do not round off individual calculations within the problem. Round only the final answer.

Now look at the following example.

Example 13

Find the area of a sheet of paper that measures 23.60 cm by 6.81 cm.

Solution:

Since one of the given measurements has four significant digits and the other has three significant digit, the answer will be rounded to three significant digits.

l (length) = 23.60 cm (four significant digits)

w (width) = 6.81 cm (three significant digits)

$$\begin{aligned} A &= lw \text{ (Area} = \text{length} \times \text{width)} \\ &= 23.60 \text{ cm} \times 6.81 \text{ cm} \\ &= 160.716 \text{ cm}^2 \\ &\doteq 161 \text{ cm}^2 \quad \text{(rounded to three significant digits)} \end{aligned}$$

Thus, the area of the sheet of paper is about 161 cm^2.

Example 14

What is the width of fence used for a rectangular yard if the area enclosed is 42.3 m^2 and the length is 7.1 m?

Solution:

Since $A = lw$

$$\begin{aligned} w &= \frac{A}{l} \\ &= \frac{42.3 \text{ m}^2}{7.1 \text{ m}} \begin{array}{l} \leftarrow \text{three significant digits} \\ \leftarrow \text{two significant digits} \end{array} \\ &= 5.9577\ldots \text{ m} \\ &\doteq 6.0 \text{ m} \quad \text{(Round to two significant digits.)} \end{aligned}$$

The **zero** must be included.

The width of the yard is about 6.0 m.

The answer must be rounded to two significant digits, since two is the least number of significant digits in either of the two values.

There is a tendency to overuse significant digits. Be reasonable in your answers if you are using a calculator to calculate answers. When using metric conversions or counting numbers, remember that they are exact numbers and have an infinite number of significant digits.

For example, the mass of three coins whose average mass is 4.73 g is $3 \times 4.73 \text{ g} = 14.2 \text{ g}$ (correct to three significant digits).

The three is an exact number since you do not count 3.01 coins. Therefore, three has an infinite number of significant digits.

15. Perform the calculations and round the answers to the correct number of significant figures.

a. Add.

$$\begin{array}{rl} 3.37 & g \\ 9.475 & g \\ \underline{21.2} & \underline{g} \end{array}$$

b. Add.

$$\begin{array}{rl} 0.05 & L \\ 1.1 & L \\ \underline{9.905} & \underline{L} \end{array}$$

c. Subtract.

$$\begin{array}{rl} 200.0 & m \\ \underline{10.67} & \underline{m} \end{array}$$

d. Subtract.

$$\begin{array}{rl} 0.055 & kg \\ \underline{0.01} & \underline{kg} \end{array}$$

e. Multiply.

(3 g/mol)(6.91 mol)

f. Multiply.

(0.035 g)(31°C)(4.19 J/g°C)

g. Divide.

$$\frac{12.6\ g}{12.01\ g/mol}$$

h. Divide.

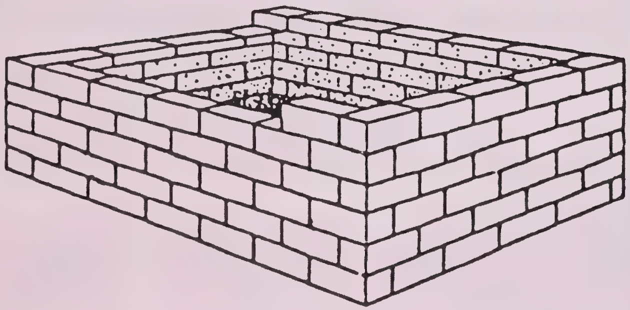
$$\frac{2.00\ g}{0.019\ g/cm^3}$$

For solutions to **Extensions**, turn to **Appendix A, Topic 4**.

Topic 5 Adding and Subtracting Polynomials

Introduction

Having learned the composition of your building blocks, you are ready to start building the wall. You are going to add and subtract those polynomial blocks.



What Lies Ahead

Throughout this topic you will learn to

1. add and subtract polynomials

Now that you know what to expect, turn the page to begin your study of adding and subtracting polynomials.

Exploring Topic 5

Activity 1

Add and subtract polynomials.

Adding and subtracting polynomials is similar to adding and subtracting objects. Look at the following situation.

Case 1

A small bowl holds three apples. Five more apples are added to the bowl. How many apples are in the bowl now?

3 apples + 5 apples = 8 apples

There are 8 apples in the bowl now.

Case 2

Four bananas are added to the apples in the bowl. What is in the bowl now?

8 apples + 4 bananas = 8 apples + 4 bananas

There are 8 apples and 4 bananas in the bowl now.

In the first case, you were able to combine the number of objects and obtain a total. These are referred to as like objects. You can add like objects.

In the second case, you were not able to combine the objects. You had to state the number of each object separately. These are referred to as unlike objects.

Think of the names of the objects as the **literal coefficients** of a term. **If the literal coefficients of a group of terms are exactly the same, then the terms are like terms.** You can add the numerical coefficients of like terms and simply repeat the literal coefficients in the sum.

$5x + 6x = 11x$
like terms

$5x^2 + 13x^2 = 18x^2$
like terms

$5x + 7x^2 = 5x + 7x^2$
unlike terms

$4y + 5x = 4y + 5x$
unlike terms

$3xy + 4x = 3xy + 4x$
unlike terms

Case 1 and Case 2 demonstrate like and unlike terms. Like terms must have exactly the same variables, and those variables must have the same exponents.

Example 1

- Simplify the polynomial $3x-5y+7y+2x$.

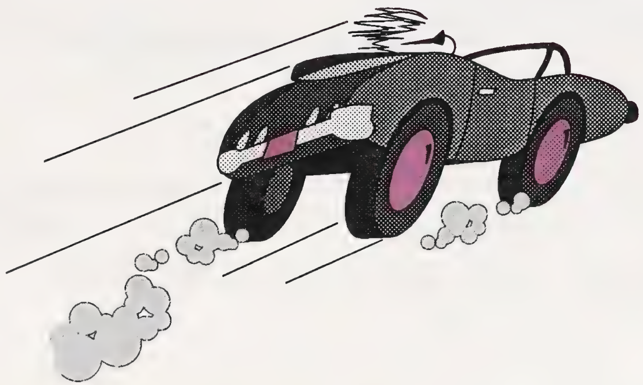
Solution:

$$\begin{aligned} 3x-5y+7y+2x &= 3x+2x-5y+7y \qquad \text{(Group like terms.)} \\ &= 5x+2y \end{aligned}$$

- Simplify $3x^2-8x-5x^2+11x$.

Solution:

$$\begin{aligned} 3x^2-8x-5x^2+11x &= 3x^2-5x^2-8x+11x \qquad \text{(Group like terms.)} \\ &= -2x^2+3x \end{aligned}$$



- Simplify $\left(3xy^2+7x^2y^2-6x^3y\right)+\left(11xy^2-4x^2y^2-x^3y\right)$.

Solution:

$$\begin{aligned}\left(3xy^2+7x^2y^2-6x^3y\right)+\left(11xy^2-4x^2y^2-x^3y\right)&=3xy^2+7x^2y^2-6x^3y+11xy^2-4x^2y^2-x^3y && \text{(Remove brackets by multiplying each term inside the brackets by +1.)}\\&=3xy^2+11xy^2+7x^2y^2-4x^2y^2-6x^3y-x^3y && \text{(Group like terms.)}\\&=14xy^2+3x^2y^2-7x^3y && \text{(Arrange in descending order.)}\\&=-7x^3y+3x^2y^2+14xy^2\end{aligned}$$

Descending Order: arranging terms with the highest exponents to the lowest

- Simplify $\left(3pq-5p^2q+2p^3q\right)-\left(3pq-7p^2q+p^3q\right)$.

Solution:

$$\begin{aligned}\left(3pq-5p^2q+2p^3q\right)-\left(3pq-7p^2q+p^3q\right)&=3pq-5p^2q+2p^3q-3pq+7p^2q-p^3q && \text{(Remove brackets by multiplying terms inside first brackets by +1 and terms inside second brackets by −1.)}\\&=3pq-3pq-5p^2q+7p^2q+2p^3q-p^3q && \text{(Group like terms.)}\\&=0pq+2p^2q+p^3q && \text{(Arrange in descending order.)}\\&=p^3q+2p^2q\end{aligned}$$

Adding and subtracting polynomials can also be done by using the vertical or **stack** method. Examine the following examples to see this method in actual use.

Example 2

Find the sum or difference of the following groups of polynomials.

- Find the sum of $3x^2 - 5x + 2$ and $5x^2 + 2x - 2$.

  Solution:

  Add.

$$\begin{array}{l} 3x^2 - 5x + 2 \\ \underline{5x^2 + 2x - 2} \\ 8x^2 - 3x \end{array} \quad \text{(Line up like terms.)}$$

- Find the difference of $5x^2y - 3xy - 2y^2 + 4$ and $3x^2y - 3xy + 2y^2 - 5$.

  Solution:

  Subtract. Remember to change the signs in the second polynomial because you are subtracting.

$$\begin{array}{l} 5x^2y - 3xy - 2y^2 + 4 \\ \underline{3x^2y - 3xy + 2y^2 - 5} \\ 2x^2y + 0xy - 4y^2 + 9 = 2x^2y - 4y^2 + 9 \end{array}$$

- Find the sum of $3x^2 + 3xy - 5$ and $3xy - 5y^2 + 2$.

  Solution:

  Add.

$$\begin{array}{rrrr} 3x^2 & +3xy & & -5 \\ & 3xy & -5y^2 & +2 \\ \hline 3x^2 & +6xy & -5y^2 & -3 \end{array}$$

Notice how the like terms are lined up. The empty spaces can be regarded as zeros, namely $0x^2 = 0$ and $5y^2 = 0$.

Notice that the like terms are lined up in the stack method. This has the same effect as grouping like terms together. It makes it easier to add the numerical coefficients when terms are arranged in this way.

By adding and subtracting polynomials, you can make the task of evaluating an expression much simpler.

Example 3

If $P(x)=(3x^2-7x+9)-(2x^2+6x+9)$, evaluate $P(-3)$.

Solution:

$$\begin{aligned} P(x) &= (3x^2-7x+9)-(2x^2+6x+9) \\ &= 3x^2-7x+9-2x^2-6x-9 \\ &= x^2-13x \end{aligned}$$

$$\begin{aligned} \therefore P(-3) &= (-3)^2-13(-3) \\ &= 9+39 \\ &= 48 \end{aligned}$$

If you evaluate without simplifying, the question becomes much more complicated.

$$P(x)=(3x^2-7x+9)-(2x^2+6x+9)$$

$$\begin{aligned} P(-3) &= [3(-3)^2-7(-3)+9]-[2(-3)^2+6(-3)+9] \\ &= 3(9)+21+9-[2(9)-18+9] \\ &= 27+21+9-(18-18+9) \\ &= 27+21+9-18+18-9 \\ &= 75-27 \\ &= 48 \end{aligned}$$

As you can see the final value is the same in both cases.

It is time to do the questions that follow.

Do questions 1, 2, and 3. Do any three of questions 4, 5, 6, and 7.

1. Group the like terms from the following lists.

   a. $7a$, $6b^2$, $9ap$, ap^2, $12b^2$, $3ap^2$, $8a$, $3ap$, $13a$, $13b^2$

   b. 13, $4x$, 7p, $5pw^2$, 2, $5p$, $23pq$, $6x$, 4, $25x$

2. Simplify each of the following.

   a. $5x-3y+6x+5y$

   b. $4z+5c-7z^2+7c$

   c. $6d^2+4d-3+5d^2-3d+1$

   d. $(4e^4-5e^2+3)-(e^4-3e^2+6)$

   e. $f^3+17+(f^2-5f+2)+(3f^3-f^2-3)$

3. Perform the following operations.

a. Add the following.

$$\begin{array}{r} 4x^2 - 3x + 4 \\ \underline{2x^2 + 5x - 7} \end{array}$$

b. Add the following.

$$\begin{array}{l} 3x^2 - 3y + x - 14 \\ \underline{2x^2 + 2y^2 + 3x + 9} \end{array}$$

c. Subtract the following.

$$\begin{array}{l} 3n^2 - 4n + 13 \\ \underline{5n^2 + n - 8} \end{array}$$

d. Subtract the following.

$$\begin{array}{l} 3m^2 + 5w^2 - 3w + 24 \\ \underline{3m^2 - 6m + 4w - 12} \end{array}$$

4. Simplify, and then evaluate when $p = -1$, $q = 4$, and $r = 2$:

a. $(p+q+r)-(p-q-r)$

b. $(2p+q-3r)+(3q-r-2p)$

c. $(p^2 - 2pq + q^2) - (3p^2 + 3pq - 3q^2)$

d. $(pqr + p^2qr^2) - (3pqr - 3p^2qr^2) + (2pqr + 4p^2qr^2)$

5. Explain what conditions must exist for two terms to be added.

6. What advantages does the stack method of additon have over the normal form of addition?

7. A small printing company requires that its product go through three departments. The cost of the product going through the three departments is shown in the chart. Write a polynomial that will show the total cost of the product.

Department	Cost per person (p)	Cost per hour (h)
Department 1	\$16	\$9
Department 2	\$9	\$2
Department 3	\$22	\$27

For solutions to **Activity 1**, turn to **Appendix A, Topic 5**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

Try the following puzzle below.

Simplify each expression. Find your answer in the answer column and notice the letter next to it. Write this letter in each box that contains the number of that exercise at the bottom of the next page.

Why Did the Donkey Get a Passport?[1]

1. $8x^2+2x-5x+7$
2. $4-3x^2-9x-7+x^2$
3. $-5x+8-4x^2-4x+2x^2$
4. $x^2-(-3x)+4+7x^2-8x-6$
5. $-x-5x+\left(-3x^2\right)-9-2x+7$
6. $-7+x^3-5x^2+4x-5x+3$
7. $4x^3+6x^2+6x-1+5x^3-x^2-(-9)$
8. $-7x+5x^2-5x^3+8x+3x^2-7x^3+x^3$
9. $6x^3+(-2)-(-2x)-5x^3-4x^2+x+4x^2+15$
10. $6x^5-2x^4+6x^3-12x^5-6x^4+9x^3$
11. $8ab-3b^2+2a^2-4ab+4b^2$
12. $5a^2b+9ab^2-2a^2b-13ab^2$
13. $3a^3+b^3-6a^2b-a^3+6ab^2+a^2b$
14. $a^2b^2+a^2b-a^3-ab^2+a^2b-b^3-a^2b^2-b^3$

[1] From Algebra With Pizzazz Steve and Janis Marcy *Creative Publications*

C $-11x^3 + 8x^2 + x$

N $-6x^5 - 7x^4 + 9x^3$

E $8x^2 - 5x - 2$

V $3a^2b - 4ab^2$

L $8x^2 - 3x + 7$

K $2a^3 - 5a^2b - ab^2 - 2b^3$

H $x^3 + 3x + 13$

U $x^3 - 5x^2 - x - 4$

B $2a^2 + 4ab + b^2$

A $-2x^2 - 9x - 3$

O $2a^3 - 5a^2b + 6ab^2 + b^3$

M $9x^3 + 5x^2 + 6x + 8$

S $-2x^2 - 9x + 8$

T $-6x^5 - 8x^4 + 15x^3$

R $-a^3 + 2a^2b - ab^2 - 2b^3$

D $-3x^2 - 8x - 2$

3	13	9	4	8	13	6	1	5	11	4	8	13	7	4	2	10	14	2	12	4	1	11	6	14	14	13

For solutions to **Extra Help**, turn to **Appendix A, Topic 5**.

Extensions

Try the following puzzle.

Daffynition Decoder[1]

For each exercise, subtract the second polynomial from the first. Find your answer in the answer column and notice the letter next to it. Each time the exercise number appears in the code, write this letter in the space provided. Keep working and you will decode the "de-fun-itions."

1. Romantic: ___ 11 ___ 13 ___ 8 ___ 12 ___ 11 ___ 1 ___ 8 ___ 11 ___ 13 ___ 8 ___ 13 ___ 10 ___ 3 ___ 5 ___ 12

2. American: ___ 11 ___ 2 ___ 11 ___ 9 ___ 9 ___ 6 ___ 5 ___ 7 ___ 13 ___ 12 ___ 11 ___ 8 ___ 13 ___ 3 ___ 4

(1) $(7x+4)-(2x+9)$

(2) $(3x+12)-(5x-6)$

(3) $(-4x^2+10)-(6x^2-9)$

(4) $(2x^2+3x+8)-(x^2+5x-1)$

(5) $(-x^2+9x-2)-(9x^2-4x+4)$

(6) $(3x^2 7x+1)-(8+5x+x^2)$

(7) $(4x^3+6x^2-8x)-(x^3-2x^2+12x)$

(8) $(x^3+2x^2+5x)-(3x^2-x-7)$

(9) $(x^4+8x^2-1)-(x^2-3x^3+x^4)$

(10) $(5x^4-2x^2)-(3x-2x^2-4x^3+6x^4)$

(11) $(3x^2+7xy-2y^2)-(x^2-6xy+2y^2)$

(12) $(-x^2-9xy+5y^2)-(4x^2-2xy-y^2)$

(13) $(4x^2y-3xy^2)-(3x^2y-8xy^2)$

[1] From Algebra With Pizzazz Steve and Janis Marcy *Creative Publications*

M $-x^4+4x^3-7x^2$

S $-x^4+4x^3-3x$

U $3x^3+5x^2+7$

L $5x-5$

E $-10x^2+19$

F $2x^2+2x-19$

C $-10x^2+13x-6$

H $-2x+18$

T $-5x^2-7xy+6y^2$

O $3x^3+8x^2-20x$

P $3x^3+7x^2-1$

R x^2-2x+9

A $2x^2+13xy-4y^2$

N x^2y+5xy^2

Y $2x^2+2x-7$

B $-5x^2-6xy+7y^2$

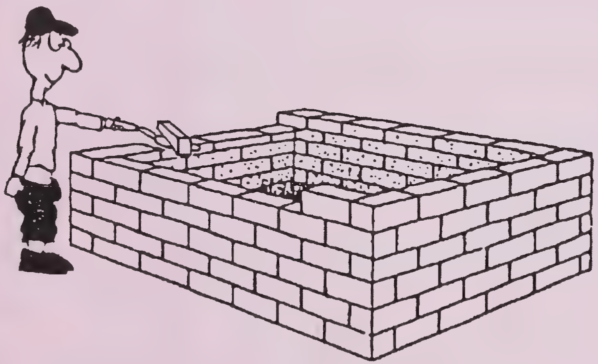
I x^3-x^2+6x+7

For solutions to **Extensions**, turn to **Appendix A**, **Topic 5**.

Topic 6 Multiplying Polynomials

Introduction

What is the next building block? You are going to extend the building block of multiplication of polynomials.



What Lies Ahead

Throughout this topic you will learn to

1. multiply polynomials by monomials
2. multiply binomials by binomials

Now that you know what to expect, turn the page to begin your study of multiplying polynomials.

Exploring Topic 6

Activity 1

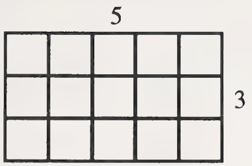
Multiply polynomials by monomials.

You are going to use the knowledge you developed in finding areas of rectangles and squares to develop the method for finding the product of two polynomials.

The area of a rectangle is found by multiplying its length by its width. The following two cases show how you used the area formula to find the areas of rectangles.

Case 1

Area of a Rectangle:

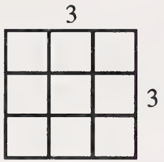


$$\begin{aligned} A &= l \times w \\ &= 5 \text{ units} \times 3 \text{ units} \\ &= 15 \text{ square units} \\ &= 15 \text{ u}^2 \end{aligned}$$

Case 2

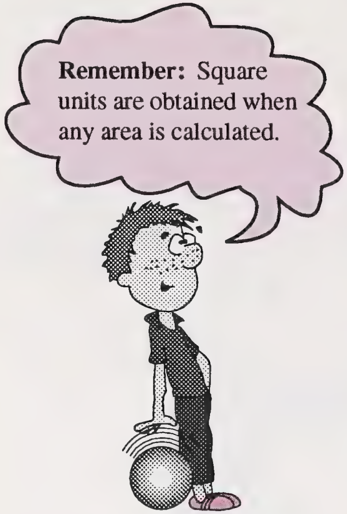
The second case deals with a special kind of rectangle called a square. Here the length and width are equal quantities. This is denoted in a special formula for the square, s times s.

Area of a Square:



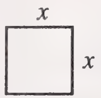
$$\begin{aligned} A &= l \times w \\ &= s \times s \\ &= 3 \text{ units} \times 3 \text{ units} \\ &= 9 \text{ square units} \\ &= 9 \text{ u}^2 \end{aligned}$$

These same principles can be used to find the areas of rectangles and squares that have measures of variables and single units along their sides.



For finding the product of two polynomials, you will only have three different types of areas. These three types are shown here.

Type 1

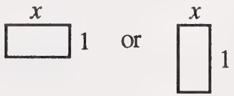


$A = l \times w$

$= x \times x$

$= x^2$

Type 2



$A = l \times w$

$= x \times 1$

$= x$

Type 3

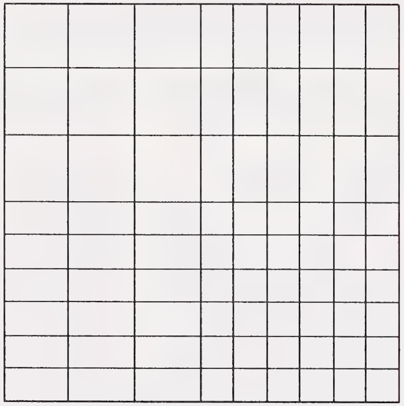
$A = l \times w$

$= 1 \times 1$

$= 1$

These are the only types of areas that you can have on the binomial grid.

A typical binomial grid is shown next.



The binomial grid has limitations, but it will help you develop a procedure for multiplying polynomials. The limitations are as follows:

- can only multiply two factors
- can only use factors that are binomial or monomial
- can only use one type of variable in the two factors

Now you are ready to begin working with **binomial grids**. The following example will demonstrate how to use a binomial grid to perform a multiplication of polynomials.

Note: Any variable can be used in place of x.

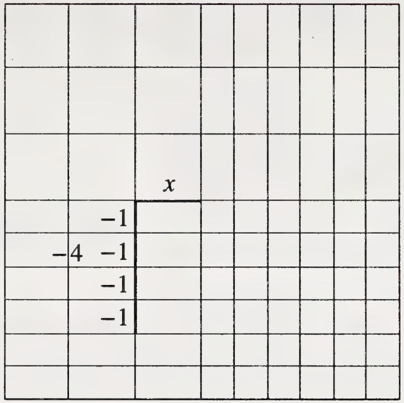
All of the areas are in square units.

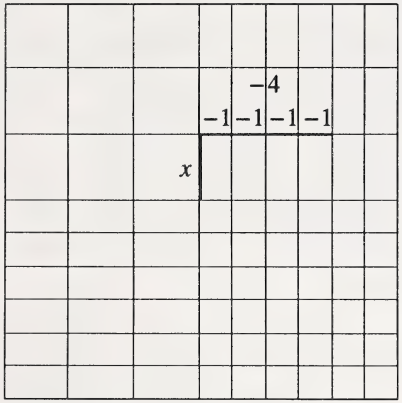
Example 1

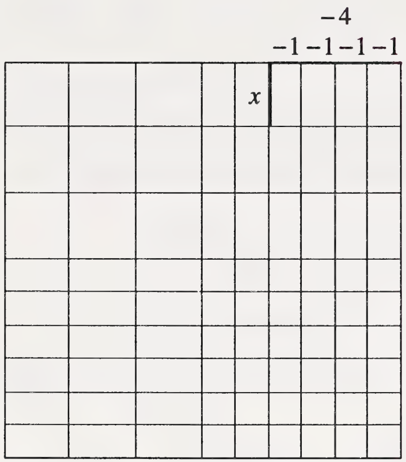
Find the product of $(-4)(x)$.

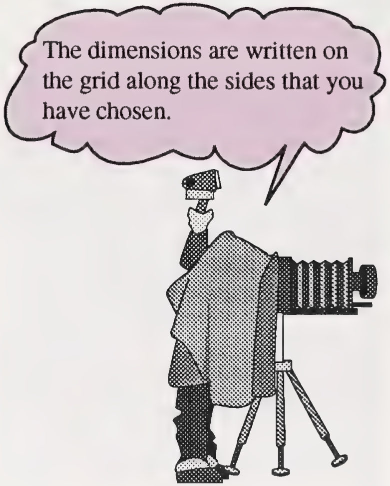
Solution:

The factors -4 and x are used as the dimensions of a rectangle. Find any place in the grid where you have these dimensions together. Three different places on the grid are shown. All of these are considered correct. There are many other correct locations or placements possible.

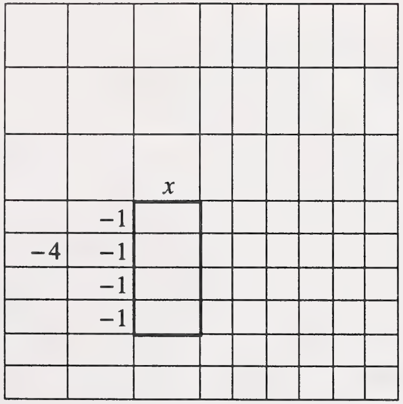




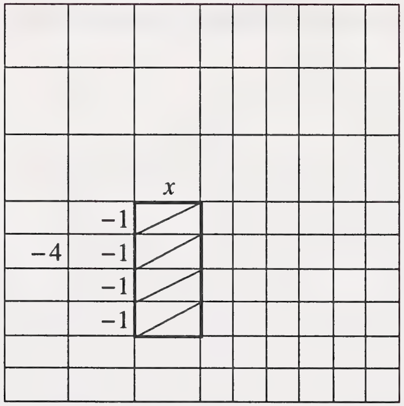




Next, use the dimensions that you have selected and enclose the entire rectangle. In this example there are four x-rectangles enclosed.



Now look for all negative dimensions. Start with the dimensions along the top of the rectangle. From each negative dimension, slash down through the entire depth of the rectangle. Next go to the dimensions along the left side of the rectangle. If there are negative dimensions, slash the other way to the end of the rectangle. **Single-slashed** squares represent negative products. **Double-slashed** squares represent positive products from negative factors.



In this particular example, each square was only slashed once. There are four slashed x-rectangles with negative or single slashes. Therefore, the product is $-4x$.

$$(-4)(x) = -4x$$

It appears from this solution that a short cut can be taken algebraically. Examine the next example and see if you can predict the short cut.


No slashes means positive.


or

Single-slashed means negative.

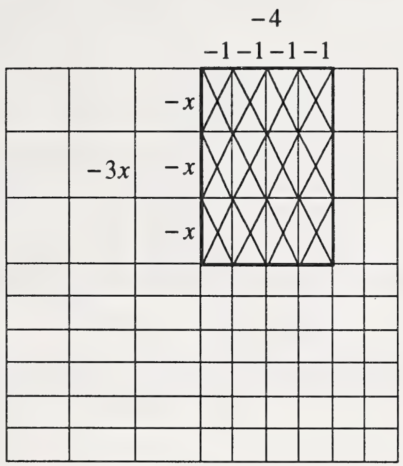

Double-slashed means positive.

Example 2

Find the product of $(-3x)(-4)$.

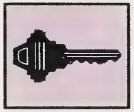
Solution:

Here the squares are double-slashed since you had to slash down and to the right a second time. Therefore, all of these rectangles are positive.



Since there are twelve positive double-slashed x-rectangles, the product is $(-3x)(-4) = 12x$.

Do you see a way you can multiply two mononials without using the grid?



To multiply two or more monomials, first multiply the numerical coefficients; then, multiply the variables following the rules of exponents.

Use the previous rule to find the solution in the following example.

Example 3

Find the following product.

$(-2x^2)(4x^3)(-3y)$

Solution:

$$\begin{aligned}(-2x^2)(4x^3)(-3y) &= [-2\times 4\times(-3)](x^2\times x^3)(y)\\ &= (+24)(x^{2+3})(y)\\ &= 24x^5y\end{aligned}$$

First regroup the coefficients; then perform the multiplications.

What rule do you use to multiply a binomial by a monomial?

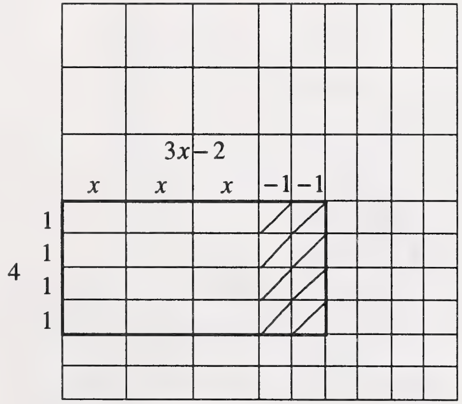
Examine how to do this type of multiplication by studying another example.

Example 4

- Find the product of $4(3x-2)$.

Solution:

In this grid the defined area has eight negative (slashed) 1-squares and twelve positive (nonslashed) x-rectangles. Therefore, $4(3x-2)=12x-8$.



To arrive at the solution from the written question, it appears that both the $3x$ and the -2 are being multiplied by the four. This would make sense because it would be following the steps of the distributive property.

That is, $4(3x-2)=4(3x)-4(2)$

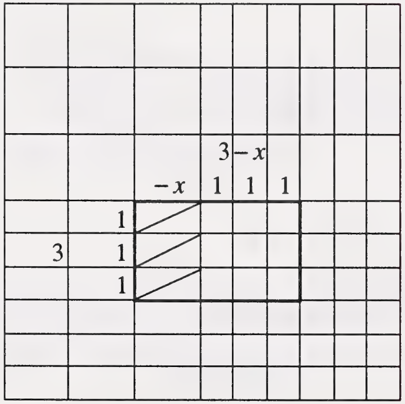
$=12x-8$

Try the second part of this example to see if this holds true.

- Find the product of $3(3-x)$.

Solution:

This grid has three negative (slashed) x-rectangles and nine positive (nonslashed) 1-squares within the boundary. Therefore, $3(3-x)=9-3x$.



Applying the distributive property holds true for this example.

That is, $3(3-x)=3(3)-3(x)$

$=9-3x$

Note: Distributive Property

$a(b+c)=ab+ac$ or

$x(y-z)=xy-xz$

When multiplying a binomial by a monomial, multiply both terms of the binomial by the monomial.

$$a(b+x)=ab+ax$$

Now try using this rule in the next example.

Example 5

Find the product of $5(5y-3)$.

Solution:

$$\begin{aligned}5(5y-3)&=5(5y)-5(3)\\&=25y-15\end{aligned}$$

Now put this information into practice as you do the following questions.

1. Find the following product using a binomial grid. Binomial grids are provided in **Appendix B**.

   a. $(2x)(3x)$ b. $(3x)(-x)$

   c. $x(3x-4)$ d. $2x(-2x+3)$

   e. $-5(2x+3)$ f. $-2(-3x-5)$

2. Find the following products algebraically.

   a. $(3x^2)(5x)$ b. $(15a^3)(2a)$

   c. $(5y^3)(8y^2)(2y)$ d. $(5w^4)(12w^5)(w^2)$

   e. $(6y^2)(3xy)(5x^2y^3)$ f. $(18xy^2)(9x^2y)(3xy)$

   g. $5s(s-4)$ h. $3t(9t-4)$

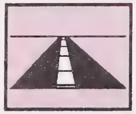
   i. $4t(t^2+2s)$ j. $2m(3m^3-5n^2)$

   k. $4t(3t^2-16t+13)$ l. $12h(3h^2-5h+3)$

3. Explain how to multiply a polynomial by a monomial.

For solutions to **Activity 1**, turn to **Appendix A**, **Topic 6**.

Activity 2



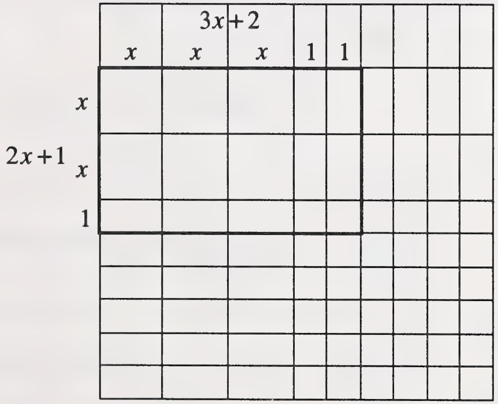
Multiply binomials by binomials.

You can use the same procedure as in Activity 1 to develop a rule for multiplying a binomial by a binomial. See how the binomial grid is used in the following examples.

Example 6

- Find the product of $(2x+1)(3x+2)$.

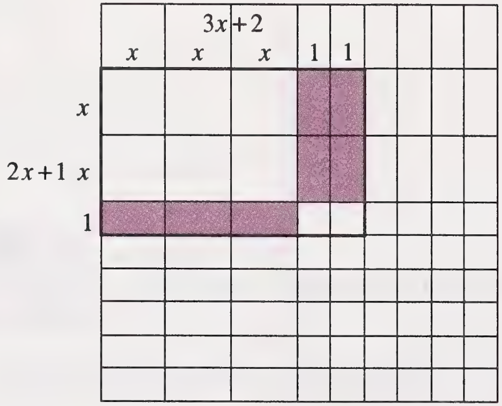
Solution:



This grid contains six positive x^2-squares, seven positive x-rectangles, and two positive 1-squares. Therefore,

$(2x+1)(3x+2)=6x^2+7x+2$.

It is easy to see where the $6x^2$ $(3x \times 2x)$ and the $2(2\times1)$ come from, but what about the $7x$?

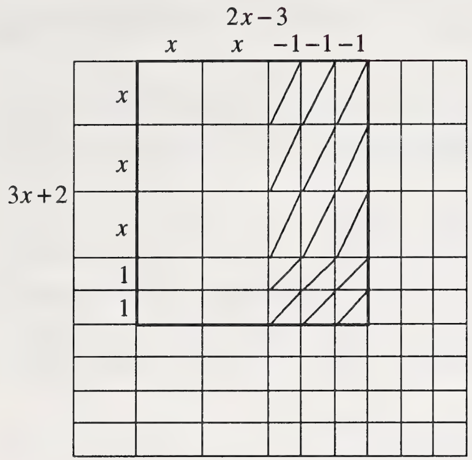


Notice that the coloured x-rectangles are in two groups: one group of four and another group of three. The group of four is represented by the multiplication of $2x$ and 2, and the group of three is represented by the multiplication of $3x$ and 1. This means that both terms of the second binomial are multiplied by each term in the first binomial.

See if this holds true for the rest of the examples.

- Find the product of $(3x+2)(2x-3)$.

Solution:



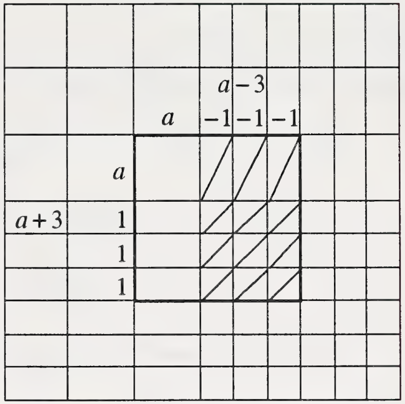
In this grid there are six positive x^2-squares, four positive x-rectangles, nine negative (slashed) x-rectangles, and six negative (slashed) 1-squares. Therefore,

$$\begin{aligned}(2x-3)(3x+2) &= 6x^2+4x-9x-6\\ &= 6x^2-5x-6\end{aligned}$$

The explanation for the first example also holds true for this example.

- Find the product $(a+3)(a-3)$.

Solution:



In this grid there are sixteen tiles. One is a positive a^2-square, three are positive a-rectangles, three are negative (slashed) a-rectangles, and nine are negative (slashed) 1-squares.

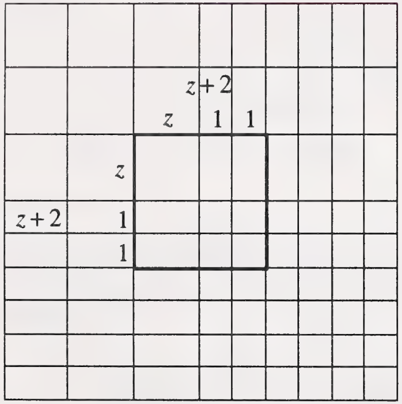
Therefore, $(a+3)(a-3) = a^2+3a-3a-9$

$$\begin{aligned}&= a^2+0a-9\\ &= a^2-9\end{aligned}$$

This is a special type of binomial multiplication. Notice that the only difference between the first and second binomial is the sign on the second term. A binomial product that results from this rule is called a **Difference of Squares Binomial**. To find the product quickly, square the first term, square the second term, and take the difference of the two products.

- Find the product of $(z+2)(z+2)$.

Solution:



In this grid you will find one positive z^2-square, four positive z-rectangles, and four positive 1-squares.

Therefore, $(z+2)(z+2) = z^2 + 2z + 2z + 4$

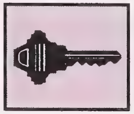
$$= z^2 + 4z + 4$$

This is also a special type of binomial multiplication. Notice both binomials are exactly the same. This product is called a **Perfect Square Trinomial**.

Did the multiplication rule hold true in the previous examples?

The algebraic method of multiplying a binomial by a binomial is called the **FOIL** method.

First terms, **O**utside terms, **I**nside terms, **L**ast terms



Multiply the **first terms** of both binomials; multiply the **two outside terms** of the binomials; multiply the **two inside terms** of the binomials; and then multiply the **last terms** of the binomials. Add all like terms and link all the terms using plus or minus signs as they apply.

$$\begin{aligned} (2x-3)(3x+2) &= \overset{F}{(2x)(3x)} + \overset{O}{(2x)(2)} - \overset{I}{(3)(3x)} - \overset{L}{(3)(2)} \\ &= 6x^2 + 4x - 9x - 6 \\ &= 6x^2 - 5x - 6 \end{aligned}$$

Use this same rule in the next example.

Example 7

Find the product of $(2y-9)(8y-12)$.

Solution:

$$\begin{aligned} (2y-9)(8y-12) &= \overset{F}{2y(8y)} + \overset{O}{2y(-12)} - \overset{I}{9(8y)} - \overset{L}{9(-12)} \\ &= 16y^2 - 24y - 72y + 108 \\ &= 16y^2 - 96y + 108 \end{aligned}$$

Now try some exercises on your own. Do either the first or second column of questions 1, 2, and 3. Then do questions 4 and 5.

1. Find the following products using the binomial grid. Binomial grids are provided in **Appendix B**.

   a. $(y-3)(y+2)$　　b. $(a+5)(a-4)$

   c. $(y-3)(3y+4)$　　d. $(a-2)(2a+5)$

   e. $(2y+1)(3y+5)$　　f. $(3y-5)(3y+2)$

2. Find the following products algebraically.

   a. $(w-3)(w+4)$　　b. $(t+5)(t-7)$

   c. $(2q+3)(q+6)$　　d. $(2w-8)(w-4)$

   e. $(3r+4)(5r-7)$　　f. $(10m+7)(2m-5)$

   g. $(3d+7t)(d-3t)$　　h. $(7s+5t)(7s-5t)$

   i. $(3m-2n)(3m+4)$　　j. $(5n-3m)(5n+4)$

   k. $(2m^2-m)(3m^2+m)$　　l. $(6w^3+w)(2w^2+3w)$

3. Find the following products. Identify the special type of binomial that each product represents.

   a. $(x-2)(x+2)$　　b. $(b+9)(b-9)$

   c. $(3b-4)(3b+4)$　　d. $(4n^2-8)(4n^2+8)$

   e. $(x+2)(x+2)$　　f. $(2x-7)(2x-7)$

   g. $(3q-2p)(3q-2p)$　　h. $(3m^4+2n^2)(3m^4+2n^2)$

4. Simplify the following.

   a. $(3c+8)^2$

   b. $(2b-3)^2-(3b+4)(2b+3)$

   c. $(2x+y)(2x+y)-(x-2y)^2$

   d. $3d(2d-3)^2+(-5d^2+4)$

5. Explain a simple rule of how to find a product that is a perfect square trionomial.

For solutions to **Activity 2**, turn to **Appendix A**, **Topic 6**.

If you have access to a videocassette player, you may view the video program titled *Multiplying Polynomials*. This program reviews activities 1 and 2 of this topic.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

You may decide to do both.

Extra Help

Try the following puzzle.

For each exercise, multiply the polynomial by the monomial. Find the answer of each exercise and notice the letter next to it. Write this letter in the box that contains the number of that exercise.

What Did the Girl Mushroom Say About the Boy Mushroom After Their First Date?[1]

1. $5(2n^2+n)$
2. $3n(8n^2-2n)$
3. $n^2(4n-3)$
4. $-2n(4+5n^3)$
5. $-6n^2(4n^2-9)$
6. $4a(a^2-2a+3)$
7. $-2a^2(9-a-4a^2)$
8. $a^2b(a^2-b^2)$
9. $-3ab^2(a^3b^2-2a^2b)$
10. $2ab(a^2+4ab-3b^2)$
11. $x^2y(2x^2-4xy+y^2)$
12. $-2xy^2(2x^4-5x^2y^2-3y^4)$
13. $4x^3y(-x^2y+2xy-5xy^2)$
14. $-x^2y^3(7xy^3-x^2y^2+3x^3y)$
15. $3x^2y^2(2x^4y^2+3x^2y-1)$

[1] From Algebra With Pizzazz Steve and Janis Marcy *Creative Publications*

(B) $-24n^4-54n$	(H) $-18a^2+2a^3+8a^4$	(E) $-18a^2+2a^3+6a^5$
(T) $24n^3-4n$	(E) $2a^3b+8a^2b^2-6ab^3$	(E) $2a^3b+8a^2b^2-6ab^3$
(R) $-24n^4+54n^2$	(I) $-7x^3y^6+x^4y^5-3x^5y^4$	(L) $2a^3b+8ab^2-4ab$
(U) $4n^3-3n^2$	(A) $a^4b-a^2b^3$	(F) $6x^6y^4+9x^4y^3-3x^2y^2$
(S) $10n^2+5n$	(G) $4a^3-8a^2+12a$	(G) $4a^3-8a^2+12a$
(L) $24n^3-6n^2$	(W) $-18a^2+2a^3+6a^5$	(E) $-4x^5y^2+8x^4y^2-20x^4y^3$
(O) $-8n-6n^3$	(L) $-3a^4b^4+6a^3b^3$	(L) $-3a^4b^4+6a^3b^3$
(A) $-8n-10n^4$	(N) $-4x^5y^2+10x^3y^4+6xy^6$	
(M) $4a^3-8a^2+10$	(Y) $2x^4y-4x^3y^2+x^2y^3$	

7	10	1	5	13	4	9	2	11	8	15	3	12	6	14

For solutions to **Extra Help**, turn to **Appendix A, Topic 6**.

Extensions

Try the following puzzle.

For each exercise, multiply the two polynomials. Find your answer in the set of answer under the exercise. Cross out the letter beside your answer. When you finish, the answer to the title question will remain.

Why Is a Stick of Gum Like a Sneeze?[1]

1. $(x+3)(x+5)$	7. $(4a-7)(3a-2)$	13. $(n+2)(n^2+5n-3)$
2. $(x+2)(x+9)$	8. $(2a+5)(2a-5)$	14. $(3n-1)(2n^2+4n+4)$
3. $(x-8)(x+1)$	9. $(6a-1)(2a+4)$	15. $(2n+3)(6n^2-2n+1)$
4. $(x-3)(x-6)$	10. $(a+2b)(4a+b)$	16. $(4n-5)(n^2-7n-2)$
5. $(2x+9)(x-2)$	11. $(5a+3b)(a-4b)$	17. $(3n-4)(4n^2+2n+3)$
6. $(3x+1)(2x+4)$	12. $(3a-8b)(2a-b)$	18. $(n+8)(6n^2-n-4)$

[1] From Algebra With Pizzazz Steve and Janis Marcy *Creative Publications*

B	x^2-7x-8
E	$x^2+8x+15$
S	$6x^2+14x+4$
I	$6x^2+7x+4$
A	$x^2-9x+18$
U	$x^2+11x+18$
T	$x^2-13x+18$
N	$2x^2+5x-18$
T	$4a^2+9ab+2b^2$
I	$6a^2-19ab+8b^2$
S	$5a^2-11ab-12b^2$
E	$12a^2+22a-4$
R	$4a^2-25$

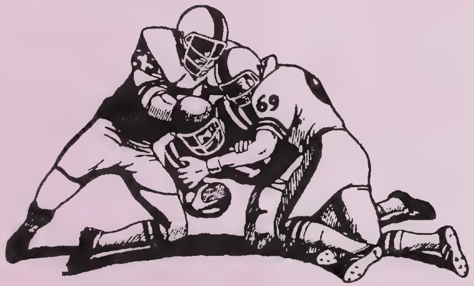
A	$4a^2+4ab+3b^2$
N	$5a^2-17ab-12b^2$
O	$12a^2-29a+14$
T	$6n^3+47n^2-12n-32$
C	$6n^3+44n^2-9n-32$
R	$4n^3-33n^2+27n+10$
I	$6n^3+10n^2+8n-4$
H	n^3+6n^2+9n-6
E	$12n^3-9n^2-2n-12$
A	$12n^3-10n^2+n-12$
N	n^3+7n^2+7n-6
W	$4n^3-30n^2+21n+10$
D	$12n^3+14n^2-4n+3$

For solutions to **Extensions**, turn to **Appendix A, Topic 6**.

Topic 7 Dividing Polynomials

Introduction

You are now prepared to tackle the building block of division of polynomials.



What Lies Ahead

Throughout this topic you will learn to

1. divide polynomials by monomials

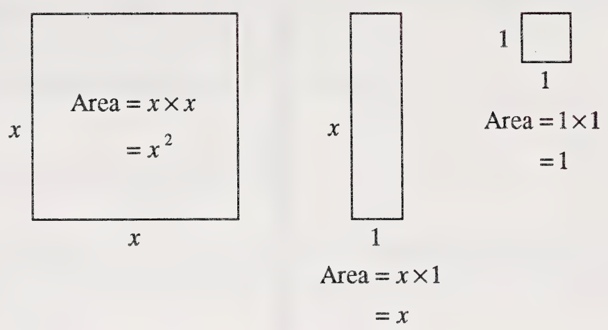
2. divide polynomials by binomials of the form $ax + b$

Now that you know what to expect, turn the page to begin your study of dividing polynomials.

Exploring Topic 7

Activity 1

Divide poynomials by monomials.



How do you divide a monomial by a monomial?

First analyse how division compares to multiplication.

factors product

$3 \times 5 = 15$

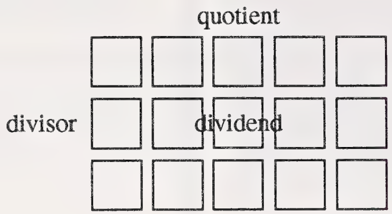
$15 \div 3 = 5$

dividend | quotient

divisor

If you examine these two equations, you will notice that the product and the dividend are the same number.

Using the algebra tiles, the division problem $15 \div 3 = 5$ can be represented as shown next. The dividend 15 is the area of the rectangle; the divisor 3 is one side of the rectangle; and the other side is the quotient 5.



Algebra Tiles

Algebra tiles consists of rectangles of three different sizes that may be used to represent polynomial products. The rectangles (or tiles) have dimensions and area as shown. The large square represents x^2; the small square represents 1; and the rectangle represents x.

Use the tiles to help you do the division. As you use the tiles, you must follow three rules:

Rule 1: Select a number of tiles whose area is equal to the dividend.

Rule 2: Create a rectangle with these tiles making sure that one side of the rectangle is equal to the divisor.

Rule 3: The other side of the rectangle will be the quotient.

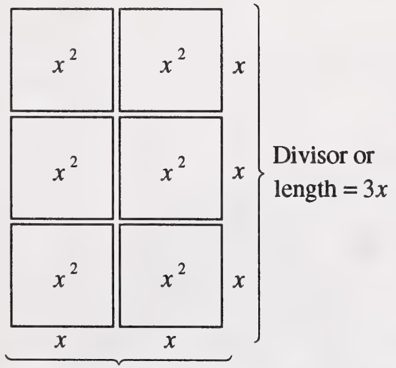
Use this idea to solve an example.

Example 1

Find the quotient for $6x^2 \div 3x$.

Solution:

Area $= 6x^2$



Quotient or width $= 2x$

Since the divisor is the length along one side of the rectangle, the quotient will be the width along the other side. Therefore, $6x^2 \div 3x = 2x$.

Now examine this division a little more closely. It appears that you can divide the 3 into the 6 and the x into the x^2. This will also give you the same answer.

Examine another question and see if this rule holds true.

Example 2

Evaluate $4y \div 2y$.

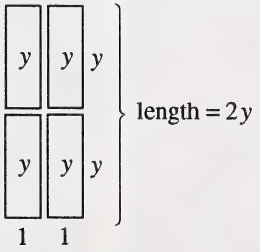
Solution:

First solve this expression using the rule.

$$\begin{aligned}4y \div 2y &= (4 \div 2)(y \div y)\\ &= (2)(y^0)\\ &= (2)(1)\\ &= 2\end{aligned}$$

Now see if the algebra tiles will give the same answer.

Dividend $= 4y$



The tiles also provide an answer of 2. The rule and the tiles result in the same answer.

Note: $y \div y = y^1 \div y^1$
$= y^{1-1}$
$= y^0$
$= 1$

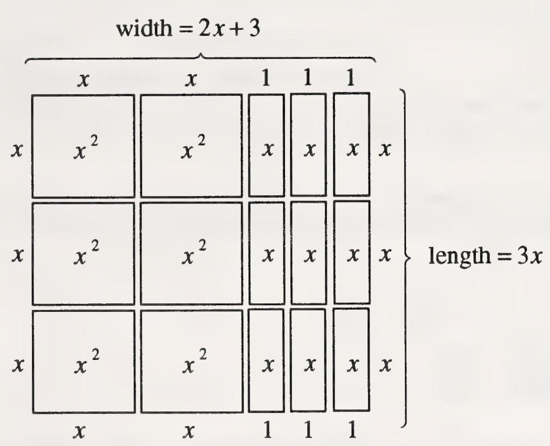
To divide a monomial into a binomial, again use the algebra tiles.

Example 3

Find the quotient of $(6x^2 + 9x) \div 3x$.

Solution:

Use the algebra tiles to represent the information. You will need six x^2-tiles and nine x-tiles. Make sure that one side of the rectangle is $3x$ units in length.



The length of the top of the rectangle is $2x + 3$. Therefore,
$(6x^2 + 9x) \div 3x = 2x + 3$.

Now take a look at another way of doing this same division problem without using the algebra tiles.

Rewrite this expression into fractional form.

$$(6x^2 + 9x) \div 3x = \frac{6x^2 + 9x}{3x}$$

Since this division has an addition in the numerator, rewrite this as the sum of two fractions with the same denominator. Now you have a monomial being divided into a monomial.

$$= \frac{6x^2}{3x} + \frac{9x}{3x}$$

Complete the two divisions.

$$= 2x + 3$$

The same solution has been achieved as with the algebra tiles.

Try solving the following division using the rule.

Example 4

Find the quotient of $\left(24m^3 + 18m^2 - 64m\right) \div 12m$.

Solution:

$$\begin{aligned}\left(24m^3 + 18m^2 - 64m\right) \div 12m &= \frac{24m^3 + 18m^2 - 64m}{12m} \\ &= \frac{24m^3}{12m} + \frac{18m^2}{12m} - \frac{64m}{12m} \\ &= 2m^2 + \frac{3m}{2} - \frac{16}{3}\end{aligned}$$

All of the division questions that have been done so far in this topic have worked out evenly. What do you do if the division does not work out evenly?

The two types of division that you have been using so far do not lend themselves well for finding remainders. You already know a method of division that gives you remainders. This is the long division you have been using for years.

Note: If the numerical coefficients do not divide evenly, divide them by their greatest common factor, and leave in $\frac{a}{b}$ form, as in Example 4.

$$\frac{18m^2 \div 6m}{12m \div 6m} = \frac{3m}{2}$$

$$\frac{64m \div 4m}{12m \div 4m} = \frac{16}{3}$$

First go over how you use long division with Natural numbers, using the expression $1562 \div 41$.

$$41\overline{)1562}$$

$$\begin{array}{r} 3 \\ 41\overline{)1562} \end{array}$$

41 goes into 156 three times.

$$\begin{array}{r} 3 \\ 41\overline{)1562} \\ 123 \end{array}$$

Multiply 41 by 3.

$$\begin{array}{r} 3 \\ 41\overline{)1562} \\ \underline{123} \\ 33 \end{array}$$

Subtract 123 frorn 156.

$$\begin{array}{r} 3 \\ 41\overline{)1562} \\ \underline{123} \\ 332 \end{array}$$

Bring down the next digit from the dividend to get 332.

$$\begin{array}{r} 38 \\ 41\overline{)1562} \\ \underline{123} \\ 332 \end{array}$$

Repeat the process. 41 goes into 332 eight times.

$$\begin{array}{r} 38 \\ 41\overline{)1562} \\ \underline{123} \\ 332 \\ 328 \end{array}$$

Multiply 41 by 8.

$$\begin{array}{r} 38 \\ 41\overline{)1562} \\ \underline{123} \\ 332 \\ \underline{328} \\ 4 \end{array}$$

Subtract 328 from 332.

Since 4 is smaller than 41, you have completed the division.

$1562 \div 41 = 38$ remainder 4, or $38\frac{4}{41}$ (express the remainder as a fraction, over the divisor).

What if the question is algebraic? Follow the same steps, as shown in the following example.

Example 5

Perform the following division.

$\left(6x^3+9x^2+2\right)\div 3x$

Solution:

$$\begin{array}{rl} 3x\big) & \overline{6x^3+9x^2+2} \end{array}$$

$$\begin{array}{rl} & 2x^2 \\ 3x\big) & \overline{6x^3+9x^2+2} \end{array}$$

$3x$ goes $2x^2$ times into $6x^3$.

$$\begin{array}{rl} & 2x^2 \\ 3x\big) & \overline{6x^3+9x^2+2} \\ & 6x^3 \end{array}$$

Multiply $3x$ by $2x^2$.

$$\begin{array}{rl} & 2x^2 \\ 3x\big) & \overline{6x^3+9x^2+2} \\ & \underline{6x^3} \\ & \quad 0 \end{array}$$

Subtract $6x^3$ from $6x^3$.

$$\begin{array}{rl} & 2x^2 \\ 3x\big) & \overline{6x^3+9x^2+2} \\ & \underline{6x^3} \\ & \quad 0+9x^2 \end{array}$$

Bring down the next term from the dividend.

$$\begin{array}{rl} & 2x^2+3x \\ 3x\big) & \overline{6x^3+9x^2+2} \\ & \underline{6x^3} \\ & \quad 0+9x^2 \end{array}$$

Repeat the process. $3x$ goes $3x$ times into $9x^2$.

$$\begin{array}{rl} & 2x^2+3x \\ 3x\big) & \overline{6x^3+9x^2+2} \\ & \underline{6x^3} \\ & \quad 0+9x^2 \\ & \qquad 9x^2 \end{array}$$

Multiply $3x$ by $3x$.

$$\begin{array}{rl} & 2x^2+3x \\ 3x\big) & \overline{6x^3+9x^2+2} \\ & \underline{6x^3} \\ & \quad 0+9x^2 \\ & \qquad \underline{9x^2} \\ & \qquad\quad 0+2 \\ & \qquad\qquad \underline{0} \\ & \qquad\qquad 2 \end{array}$$

Subtract $9x^2$ from $9x^2$.

Bring down the 2.

Since $3x$ will not go into 2, you have completed the division.

$\left(6x^3+9x^2+2\right)\div 3x = 2x^2+3x$ remainder 2, or $2x^2+3x+\frac{2}{3x}$.

(Express the remainder as a fraction over the divisor.)

Now try the exercises that follow.

Do either the first or second column of questions 1 and 2; then do questions 3 and 4.

1. Use the algebra tiles to perform the following divisions. You may use the algebra tiles provided in **Appendix B**. Cut out the pieces as required.

   a. $2x^2 \div 2x$

   b. $3x^2 \div x$

   c. $\left(4x+3x^2\right)\div x$

   d. $\left(6x^2+4x\right)\div 2x$

   e. $\left(2n^3+16n^2-10n\right)\div 2n$

   f. $\left(6p^3+15p^2q^2-9pq^3\right)\div 3q$

   g. $\dfrac{3y^2+12y-6}{3}$

   h. $\dfrac{16a^4-8a^2b^2}{8a}$

2. Use the algebraic method to perform the following divisions.

   a. $16m^3n^2 \div 12m^2n$

   b. $45st^3 \div \left(-30s^2t^2\right)$

   c. $\dfrac{36a^5b^2c^3}{18a^3bc^2}$

   d. $\dfrac{-52mn^5}{-39m^2n^2}$

3. Use long division to simplify the following questions.

   a. $\left(16x^2+12x+15\right)\div 4x$

   b. $\left(15x^2+5x+16\right)\div 5x$

4. The area of a rectangular pool is given as $\left(9x^4-12x^3+6x^2\right)\text{ m}^2$. If the width of the pool is $3x^2$ m, find the expression that represents the length of the pool.

For solutions to **Activity 1**, turn to **Appendix A, Topic 7**.

Hint: Area of rectangle $= l \times w$.

Activity 2

Divide polynomials by binomials of the form $ax+b$.

You may use the algebra tiles to develop a method for dividing by a binomial. Study the examples where both the tile method and the algebraic method are used. Study both methods together. This will help show why you do certain steps in the algebraic method.

Example 6

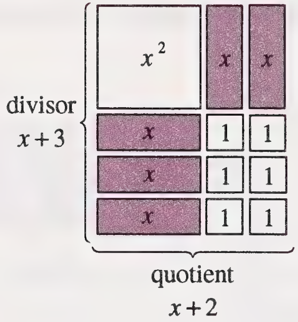
Perform the following division.

$$\left(x^2+5x+6\right)\div(x+3)$$

Solution:

Using Tiles

Select the following tiles from your set: one x^2-tile, five x-tiles, and six 1-tiles. Arrange them into a rectangle ensuring that one side is equal to the divisor $(x+3)$.



Reading the quotient from along the bottom of the rectangle you find that

$\left(x^2+5x+6\right)\div(x+3)=x+2$.

Algebraic Method

First rewrite the expression in fraction form.

$$\left(x^2+5x+6\right)\div(x+3)=\frac{x^2+5x+6}{x+3}$$

Note from the tile arrangement that $5x$ has been split into two terms, $3x$ and $2x$.
$5x=3x+2x$

Now break the original fraction into the sum of two of the fractions. Notice the numerator of each fraction will have two addends.

$$\frac{x^2+3x}{x+3}+\frac{2x+6}{x+3}$$

Recall: Addends are two or more values which are added.

Now use the distributive property on the numerator of each fraction.

$$\frac{x(x+3)}{x+3}+\frac{2(x+3)}{x+3}$$

Divide out the common numerators and denominators.

$$\frac{x\overset{1}{\cancel{(x+3)}}}{\underset{1}{\cancel{x+3}}}+\frac{2\overset{1}{\cancel{(x+3)}}}{\underset{1}{\cancel{x+3}}}=x+2$$

Therefore, $\left(x^2+5x+6\right)\div(x+3)=x+2$.

This is the same answer as you found with the tile method.

The last example used only positive addends. Now try an example that has some negative addends.

Example 7

Find the quotient for the expression

$\left(x^2-x-6\right)\div(x+2)$.

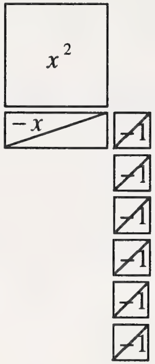
Solution:

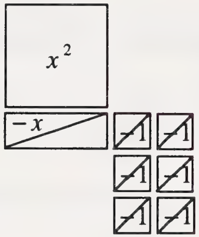
Using Tiles

You will need one x^2-tile, one x-tile, and six 1-tiles. These tiles are impossible to form into a rectangle with one side of $(x+2)$. However, adding the same number of positive and negative x-tiles will allow a rectangle to be formed.

Form the units tile into a rectangle in the lower right-hand corner. There are only two alternatives for this,

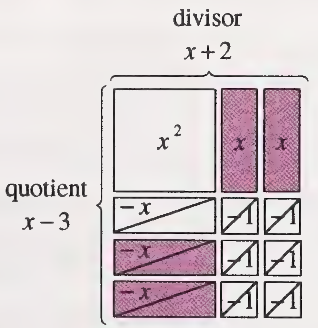
a 6 by 1 rectangle or a 3 by 2 rectangle





Recall:
$x^2-x-6=x^2+(-x)+(-6)$

Only the 3 by 2 rectangle will leave the same number of spaces (2) to be filled by x-tiles along both the top and the side. By placing two negative x-tiles along the side (next to the original x-tile) and two positive x-tiles along the top, the value of the polynomial does not change but the rectangle is completed.



Reading the quotient from the side of the rectangle, you should find that $x^2 - x - 6 \div x + 2 = x - 3$.

Algebraic Method

Write the division in fraction form.

$$\left(x^2 - x - 6\right) \div (x+2) = \frac{x^2 - x - 6}{x+2}$$

Notice from the tile method the middle term $(-x)$ has been split as $-3x$ and $2x$.

$$-x = 2x - 3x$$

Rewrite the numerator of the fractions as

$$x^2 - x - 6 = x^2 + 2x - 3x - 6$$

Now split the fraction into the sum of two fractions. Watch out for the negative signs.

$$= \frac{x^2 + 2x}{x+2} + \frac{-3x - 6}{x+2}$$

Use the distributive property in reverse on the numerators.

$$= \frac{x(x+2)}{x+2} + \frac{-3(x+2)}{x+2}$$

Divide out the common numerators and denominators.

$$= \frac{x\overset{1}{\cancel{(x+2)}}}{\underset{1}{\cancel{(x+2)}}} + \frac{-3\overset{1}{\cancel{(x+2)}}}{\underset{1}{\cancel{(x+2)}}}$$

$$= x + (-3)$$

$$= x - 3$$

Therefore, $\left(x^2 - x - 6\right) \div (x+2) = x - 3$, which is the same result as you found with the tile method.

Since it is inconvenient for you to keep using algebra tiles when dividing, you need an algebraic method of splitting the middle term of the dividend into two terms. This method is called the **sum/product** rule.

Study the steps used to find the middle terms of the expression $x^2 - x - 6$.

Step 1: The numerical coefficient of the middle term will be the sum. In $x^2 - x - 6$, the coefficient of x, the middle term, is -1. Therefore, **the sum is -1**.

Step 2: Multiply the numerical coefficient of the first term times the last term to get the **product**. In $x^2 - x - 6$, multiply the coefficient of x^2 (which is 1) times -6 (the last term). Therefore, **the product is -6**.

Step 3: Now you know that the two numbers you are looking for must have a sum of -1 and a product of -6. Make a list of all the factor pairs of -6, and find the sum of each pair.

Products	Sums
$(-1)(6) = -6$	$(-1)+6 = 5$
$(-6)(1) = -6$	$(-6)+1 = -5$
$(3)(-2) = -6$	$3+(-2) = 1$
$\mathbf{(2)(-3) = -6}$	$\mathbf{2+(-3) = -1}$

There are four factor pairs that can be multiplied to make -6, but only $2+(-3)$ equals the required sum of -1. Therefore, the coefficients of the middle terms will be $+2$ and -3.

$$x^2 - x - 6 = x^2 + 2x - 3x - 6$$

Now you can divide by $x+2$.

$$\begin{aligned} \frac{x^2 - x - 6}{x+2} &= \frac{x^2 + 2x - 3x - 6}{x+2} \\ &= \frac{x^2 + 2x}{x+2} - \frac{(3x+6)}{x+2} \\ &= \frac{x\cancel{(x+2)}^{1}}{\cancel{x+2}_{1}} - \frac{3\cancel{(x+2)}^{1}}{\cancel{x+2}_{1}} \\ &= x - 3 \end{aligned}$$

$$\therefore \left(x^2 - x - 6\right) \div (x+2) = x - 3$$

Note: If you multiply $-3x-6$ by -1, the signs change to produce $3x+6$. The -1 becomes the negative sign between the two binomial quotients.

Example 8

Use the algebraic method to find the quotient for the expression

$\frac{x^2-9x+20}{x-4}$.

Solution:

Use the sum/product rule to split the middle term $(-9x)$ into two terms.

Step 1: The sum is -9 (coefficient of middle term).

Step 2: The product is $1 \times 20 = 20$ (coefficient of first term $\times$ last term).

Step 3:

Products	Sums
$(1)(20)=20$	$1+20=21$
$(-1)(-20)=20$	$(-1)+(-20)=-21$
$(2)(10)=20$	$2+10=12$
$(-2)(-10)=20$	$(-2)+(-10)=-12$
$(4)(5)=20$	$4+5=9$
$\mathbf{(-4)(-5)=20}$	$\mathbf{(-4)+(-5)=-9}$

Therefore, the middle terms will be $-4x$ and $-5x$.

$$\begin{aligned}
\frac{x^2-9x+20}{x-4} &= \frac{x^2-4x-5x+20}{x-4} \\
&= \frac{x^2-4x}{x-4} - \frac{5x-20}{x-4} \\
&= \frac{x\overset{1}{\cancel{(x-4)}}}{\underset{1}{\cancel{x-4}}} - \frac{5\overset{1}{\cancel{(x-4)}}}{\underset{1}{\cancel{x-4}}} \\
&= x-5
\end{aligned}$$

$$\therefore \frac{x^2-9x+20}{x-4} = x-5$$

Examples 6, 7, and 8 did not have a remainder. If the quotient has a remainder, these two methods become very difficult to use. It is suggested that you use the long division method wherever there is a possibility of having a remainder.

The following example demonstrates how to use long division to divide a binomial into a polynomial.

Example 9

Find the quotient of $\left(x^3+5x^2+4\right)\div(x+2)$.

Solution:

$$x+2\overline{\smash{)}x^3+5x^2+4}$$

Notice that the x term is missing. Since all terms will be required for the division, x^3+5x^2+4 must be changed to x^3+5x^2+0x+4 to provide the missing x term.

$$x+2\overline{\smash{)}x^3+5x^2+0x+4}$$

$$x+2\overline{\smash{)}x^3+5x^2+0x+4}$$

x goes x^2 times into x^3.

$$\begin{array}{r}x^2\\ x+2\overline{\smash{)}x^3+5x^2+0x+4}\\ x^3+2x^2\end{array}$$

Multiply $x+2$ by x^2.

$$\begin{array}{r}x^2\\ x+2\overline{\smash{)}x^3+5x^2+0x+4}\\ \underline{x^3+2x^2}\\ 3x^2\end{array}$$

Subtract x^3+2x^2 from x^3+5x^2.

$$\begin{array}{r}x^2\\ x+2\overline{\smash{)}x^3+5x^2+0x+4}\\ \underline{x^3+2x^2}\\ 3x^2+0x\end{array}$$

Bring down the next term from the dividend.

$$\begin{array}{r}x^2\\ x+2\overline{\smash{)}x^3+5x^2+0x+4}\\ \underline{x^3+2x^2}\\ 3x^2+0x\end{array}$$

Repeat the process. x goes $3x$ times into $3x^2$.

$$\begin{array}{r}x^2+3x\\ x+2\overline{\smash{)}x^3+5x^2+0x+4}\\ \underline{x^3+2x^2}\\ 3x^2+0x\\ 3x^2+6x\end{array}$$

Multiply $x+2$ by $3x$.

$$\begin{array}{r}x^2+3x\\ x+2\overline{\smash{)}x^3+5x^2+0x+4}\\ \underline{x^3+2x^2}\\ 3x^2+0x\\ \underline{3x^2+6x}\\ -6x\end{array}$$

Subtract $3x^2+6x$ from $3x^2+0x$.

$$\begin{array}{r l}
 & x^2+3x \\ \cline{2-2}
x+2 \big) & x^3+5x^2+0x+4 \\
 & \underline{x^3+2x^2} \\
 & \quad 3x^2+0x \\
 & \quad \underline{3x^2+6x} \\
 & \qquad -6x+4
\end{array}$$

Bring down the next term from the dividend.

$$\begin{array}{r l}
 & x^2+3x \\ \cline{2-2}
x+2 \big) & x^3+5x^2+0x+4 \\
 & \underline{x^3+2x^2} \\
 & \quad 3x^2+0x \\
 & \quad \underline{3x^2+6x} \\
 & \qquad -6x+4
\end{array}$$

Repeat the process. x goes -6 times into $-6x$.

$$\begin{array}{r l}
 & x^2+3x-6 \\ \cline{2-2}
x+2 \big) & x^3+5x^2+0x+4 \\
 & \underline{x^3+2x^2} \\
 & \quad 3x^2+0x \\
 & \quad \underline{3x^2+6x} \\
 & \qquad -6x+4 \\
 & \qquad -6x-12
\end{array}$$

Multiply $x+2$ by -6.

$$\begin{array}{r l}
 & x^2+3x-6 \\ \cline{2-2}
x+2 \big) & x^3+5x^2+0x+4 \\
 & \underline{x^3+2x^2} \\
 & \quad 3x^2+0x \\
 & \quad \underline{3x^2+6x} \\
 & \qquad -6x+4 \\
 & \qquad \underline{-6x-12} \\
 & \qquad\qquad 16
\end{array}$$

Subtract $-6x-12$ from $-6x+4$.

Since there are no more terms to bring down, the division process is complete.

Therefore, $\left(x^3+5x^2+4\right)\div(x+2)=x^2+3x-6 \quad$ r: 16 or

$x^2+3x-6+\dfrac{16}{x+2}$.

Now try some questions that put these skills into practice.

Do either the first or second column of questions 1 and 2. Then do questions 3 and 4.

1. Use the algebra tiles provided in **Appendix B** to perfom the following divisions.

   a. $(x^2+7x+12)\div(x+4)$　　b. $(x^2+6x+5)\div(x+1)$

   c. $(x^2-5x-6)\div(x+1)$　　d. $(x^2-3x-10)\div(x+2)$

   e. $(x^2+4x-12)\div(x-2)$　　f. $(x^2-2x-8)\div(x-4)$

2. Use the algebraic method to perform the following divisions.

   a. $(x^2+10x+24)\div(x+4)$　　b. $(x^2+10x-24)\div(x-2)$

   c. $(m^2-14m+33)\div(m-3)$　　d. $(n^2-9n-52)\div(n+4)$

   e. $(a^2+10a-39)\div(a-3)$　　f. $(y^2+5y-66)\div(y-6)$

3. Use long division to simplify the following questions.

   a. $(2a^2+17a-15)\div(a+10)$　　b. $(3a^2+128a-315)\div(a+44)$

4. Explain why you would want to use long division before using the algebraic method.

For solutions to **Activity 2**, turn to **Appendix A, Topic 7**.

If you require help, do the Extra Help section.
If you want more challenging explorations, do the Extensions section.
} You may decide to do both.

Extra Help

Do the following puzzle.

Determine the following questions, and write your answer as a polynomial or mixed expression. Cross out the box containing your answer. When you finish, write the letters from the remaining boxes in the spaces at the bottom of the page.

How Did the Hunter Get Hurt While Bending Over to Study Some Tracks?[1]

1. $\dfrac{x^2+8x+15}{x+5}$
2. $\dfrac{2x^2+3x-14}{x-2}$
3. $\dfrac{x^2-5x+8}{x-3}$
4. $\dfrac{x^2-x+12}{x-6}$
5. $\dfrac{3x^2-5x-11}{x+1}$
6. $\dfrac{x^2+1+8x}{x+4}$
7. $\dfrac{x^2+4}{x-3}$
8. $\dfrac{2x^2-3x-1}{2x+1}$
9. $\dfrac{6x^2-7x+5}{3x-5}$

[1] From Algebra With Pizzazz Steve and Janis Marcy *Creative Publications*

TH $x-2+\frac{2}{x-3}$	HE $x+3+\frac{13}{x-3}$	AT $2x+2+\frac{6}{3x-5}$	ST $x+5+\frac{42}{x-6}$	SH $2x+1+\frac{10}{3x-5}$
RA $x-3+\frac{3}{2x+1}$	SK $x+3$	OT $x-2+\frac{1}{2x+1}$	IN $3x-6-\frac{5}{x+1}$	HI $x+4+\frac{9}{x-3}$
BE $3x-8-\frac{3}{x+1}$	TH $x+2-\frac{11}{x+4}$	HU $2x+7$	NT $x+4-\frac{15}{x+4}$	IM $x+7+\frac{33}{x-6}$

For solutions to **Extra Help**, turn to **Appendix A**, **Topic 7**.

Extensions

Try the following puzzle.

Determine the answers to the following questions, and write your answer as a polynomial or mixed expression. Find your answer and notice the letter next to it. Write this letter in each box that contains the number of that exercise.

What Do They Call People Who Like To Turn the Light On and Off?[1]

1. $\dfrac{4x^2-4x+3}{2x-5}$
2. $\dfrac{2x^2-20}{x+3}$
3. $\dfrac{x^3+5x^2+4x-4}{x+2}$
4. $\dfrac{1-7x^2+6x^3+17x}{3x-2}$
5. $\dfrac{x^3-8}{x-2}$
6. $\dfrac{x^3+9x^2-80}{x+4}$
7. $\dfrac{6a^2+5ab-5b^2}{2a-b}$
8. $\dfrac{a^3+4a^2b+ab^2-2b^3}{a+b}$

[1] From Algebra With Pizzazz Steve and Janis Marcy *Creative Publications*

(D) x^2+2x-7	(A) $3a+2b-\frac{8b^2}{2a-b}$	(O) $x^2+5x-18$
(T) $a^2+3ab-2b^2$	(R) $2x-6-\frac{2}{x+3}$	(C) $2x+3+\frac{18}{2x-5}$
(S) x^2+2x+4	(H) $x^2+5x-20$	(U) $2x^2-x-5+\frac{4}{3x-2}$
(N) $2x-6+\frac{7}{x+3}$	(I) x^2+3x-2	(W) $3a+4b-\frac{b^2}{2a-b}$
(E) $2x^2-x+5+\frac{11}{3x-2}$		(M) $a^2+3ab-b^2+\frac{5b}{a+b}$

5	7	3	8	1	6	6	3	8	8	4	2	5

For solutions to **Extensions**, turn to **Appendix A, Topic 7**.

Unit Summary

What You Have Learned

How is the building coming?

In this unit you tried to build on number skills to develop new skills in polynomials. You learned some new terms and new ideas about polynomials.

You added, subtracted, multiplied and divided polynomials. To perform these operations, you also had to review the rules of exponents and extend those rules to some new rules of exponents.

You will use these building blocks of polynomials in many areas of math and science for years to come.

The building of polynomials is not finished yet. In future courses you may learn to expand on these skills and keep building and building on and on.

You are now ready to complete the **Unit Assignment**.

Appendices

Appendix A
Solutions

Review

Topic 1 Polynomial Terminology

Topic 2 Evaluating Polynomials

Topic 3 Power Laws for Exponents

Topic 4 Zero and Negative Exponents

Topic 5 Adding and Subtracting Polynomials

Topic 6 Multiplying Polynomials

Topic 7 Dividing Polynomials

Appendix B
Grids and Tiles

Binomial Grids

Algebra Tiles

Appendix A
Solutions

Review

1. a. $3^4 = 3 \times 3 \times 3 \times 3$
$= 81$

The $\boxed{x^y}$ or $\boxed{y^x}$ key on your calculator can be used.

Enter	Display
3	3
$\boxed{x^y}$	3
4	4
$\boxed{=}$	81

b. $3^2 \times 3^4 = 3^{2+4}$
$= 3^6$
$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 729$

Enter	Display
3	3
$\boxed{x^y}$	3
6	6
$\boxed{=}$	729

c. $12^6 \div 12^5 = 12^{6-5}$
$= 12^1$
$= 12$

d. $\left(2^5\right)^2 = 2^{5\times 2}$
$= 2^{10}$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 1024$

Enter	Display
2	2
$\boxed{x^y}$	2
10	10
$\boxed{=}$	1024

e. $(3^2 \times 2^3)^3 = (3^2)^3 \times (2^3)^3$

$= 3^{2\times3} \times 2^{3\times3}$

$= 3^6 \times 2^9$

$= 729 \times 512$

$= 373\,248$

Enter	Display
3	3
$\boxed{x^y}$	3
6	6
$\boxed{\times}$	729
2	2
$\boxed{x^y}$	2
9	9
$\boxed{=}$	373248

f. $\frac{5^3}{5^2} = 5^{3-2}$

$= 5^1$

$= 5$

g. $\left(\frac{3^2}{5^3}\right)^3 = \frac{(3^2)^3}{(5^3)^3}$

$= \frac{3^{2\times3}}{5^{3\times3}}$

$= \frac{3^6}{5^9}$

$= \frac{729}{1\,953\,125}$

Enter	Display
3	3
$\boxed{x^y}$	3
6	6
$\boxed{=}$	729

Enter	Display
5	5
$\boxed{x^y}$	5
9	9
$\boxed{=}$	1953125

h. $3^2 + 4 \times 3 = 9 + 4 \times 3$

$= 9 + 12$

$= 21$

i. $3^2 + 3^2 - 2^3 = 9 + 9 - 8$

$= 18 - 8$

$= 10$

j. $4\left(15-3^2\right)\div 8+2=4(15-9)\div 8+2$

$$=4(6)\div 8+2$$
$$=24\div 8+2$$
$$=3+2$$
$$=5$$

2. a. $210\,000\,000=2.1\times 100\,000\,000$

$$=2.1\times 10^8$$

b. $0.000\,000\,060\,73=6.073\times 0.000\,000\,01$

$$=6.073\times 10^{-8}$$

These can also be done using the following method.

a. Move the decimal point to the right of the 2. Count the number of positions the decimal point moved. This number becomes the exponent of the 10. This exponent is positive since the number is larger than 1.

b. Move the decimal point to the right of the 6. Count the number of positions the decimal point moved. This number becomes the exponent of the 10. This exponent is negative since the number is smaller than 1.

3. a. $4.50\times 10^6=4.50\times 100\,000$

$$=4\,500\,000$$

b. $3.0014\times 10^{-7}=3.0014\times 0.000\,000\,1$

$$=0.000\,000\,300\,14$$

4. One or more factors will be between as set of parentheses. The parentheses will have an exponent. The factors may or may not have exponents.

The exponent on the parentheses is placed on each of the factors inside of the parentheses. If the factor already has an exponent, then the factor will have a new exponent: the product of the two exponents.

5. Scientific notation permits you to write a number that has a large number of digits in short form. This saves you time in writing out the number and space to write the number in.

Exploring Topic 1

Activity 1

Identify term, variable, factor, monomial, binomial, trinomial, polynomial, numerical coefficient, literal coefficient, degree, exponent, base, and power in the study of polynomials.

1. The degree is the sum of the exponents of every variable in the term.

a. $2a$
↑
1

The degree is 1.

b. $3d^2m^3n$

$\uparrow\ \uparrow\ \uparrow$

$2+3+1=6$

The degree is 6.

c. $36xy^3$

$\uparrow\ \uparrow$

$1+3=4$

The degree is 4.

d. $19m^2np^0$

$\uparrow\ \uparrow\ \uparrow$

$2+1+0=3$

The degree is 3.

e. 11

$\uparrow$

0

The degree is 0. A constant term is always degree zero.

f. $21a^3b^2cq^2$

$\uparrow\ \uparrow\ \uparrow\ \uparrow$

$3+2+1+2=8$

The degree is 8.

2. a. $3a^2b+6ab^2+b^3$

$3 \quad 3 \quad 3$

The degree of the polynomial is 3.

b. $4x^2y^3+3xy+4y+34$

$5 \quad 2 \quad 1 \quad 0$

The degree of the polynomial is 5.

c. $16p^5q^3+12p^4q^3+9p^4q^2+8pq$

$8 \quad 7 \quad 6 \quad 2$

The degree of the polynomial is 8.

d. $2m^4n^2+2m^3n^6+3m^2n^6+2mn^5$

$6 \quad 9 \quad 8 \quad 6$

The degree of the polynomial is 9.

The degree of a polynomial is the degree of the term that has the highest sum of exponents.

3.

	Expanded Form	Simplified Form	Standard Form
a.	$3\times3\times3\times3$	3^4	81
b.	$(-4)(-4)(-4)(-4)(-4)$	$(-4)^5$	-1024
c.	31×31	31^2	961
d.	$612\times612\times612$	612^3	229 220 928

4.

	Polynomial	Coefficient of x	Constant Term
a.	yes	2, 5	3
b.	yes	3, 9	0
c.	no	4, -4	0
d.	yes	-8	0

5. a. The variable of the last term is not in the numerator.

   b. The variable in the second term does not have a whole number exponent.

   c. The variable in the second term does not have a whole number exponent.

   d. This expression is not in simplified form. If it were, the variable in the last term would not have a whole number exponent $\left(\frac{8n^{-1}}{3}\right)$, or the last term would have a variable in the denominator, $\left(\frac{8}{3n}\right)$.

6. a. $3mn+3m^2n+3m^2n^2 = 3m^2n^2+3m^2n+3mn$

   b. $st^2+2s^2t+3s = 2s^2t+st^2+3s$

   c. $6a+7b^3c+2b^2c^2 = 6a+7b^3c+2b^2c^2$

   d. $x^2+2y^2+3xy = x^2+3xy+2y^2$

Activity 2

Classify polynomials according to degree, number of terms, and number of variables.

1. a. 15
   0 — constant polynomial

   b. $3y+3$
   1 0 — linear polynomial

   c. $16x^2+8x+4$
   2 1 0 — quadratic polynomial

   d. x^3-5
   3 0 — cubic polynomial

   e. $16x^2yz+12xy^2z+3y^3z$
   4 4 4 — quartic polynomial

   f. $22x^2+23y^2+11z^2$
   2 2 2 — quadratic polynomial

2. a. 1 term, monomial
   b. 2 terms, binomial
   c. 3 terms, trinomial
   d. 2 terms, binomial
   e. 3 terms, trinomial
   f. 3 terms, trinomial

3. a. no variables
   b. one variable
   c. two variables
   d. one variable
   e. three variables
   f. three variables

Extra Help

Column A	Column B	
1. $3x^2 + 2y - 3x + 4$	5	This term is of degree 7.
2. 2^5	10	This polynomial is a binomial.
3. $5x^2y^2$	7	This polynomial is a trinomial.
4. 4^6	9	This polynomial is a monomial of degree 3.
5. $x^2y^3z^2$	1	This polynomial has four terms.
6. 2^4	2	This power has an exponent of 5.
7. $x^2 - 5x - 6$	4	This power has a base of 4.
8. $6x^2$	6	This power has a value of 16.
9. $5x^3$	3	The literal coefficient of this term is x^2y^2.
10. $2x^2 - 3x$	8	The numerical coefficient of this term is 6.
11. $17x^{-2}$	11	This expression is not a polynomial.

Extensions

Column A	Column B	
1. $5x^4 + 2xyz$	4	This is a binomial of degree 4 in two variables.
2. $3x^3 + y^2$	5	This is a binomial of degree 3 in three variables.
3. $5xy - 3z$	1	This is a binomial of degree 4 in three variables.
4. $3x^2y - 4xy^3$	2	This is a binomial of degree 3 in two variables.
5. $5x^3 + 4yz$	3	This is a binomial of degree 2 in three variables.
6. $3xy - y$	6	This is a binomial of degree 2 in two variables.

Exploring Topic 2

Activity 1

Evaluate a polynomial for given values of a variable.

1. a. $$\begin{aligned} x+7 &= (4)+7 \\ &= 11 \end{aligned}$$

b. $$\begin{aligned} x-11 &= (4)-11 \\ &= -7 \end{aligned}$$

c. $$\begin{aligned} 3x-4 &= 3(4)-4 \\ &= 12-4 \\ &= 8 \end{aligned}$$

d. $$\begin{aligned} 16-2x &= 16-2(4) \\ &= 16-8 \\ &= 8 \end{aligned}$$

e. $$\begin{aligned} 5x^2-32 &= 5(4)^2-32 \\ &= 5(16)-32 \\ &= 80-32 \\ &= 48 \end{aligned}$$

f. $$\begin{aligned} 5x^2-3x+15 &= 5(4)^2-3(4)+15 \\ &= 5(16)-12+15 \\ &= 80-12+15 \\ &= 83 \end{aligned}$$

g. $$\begin{aligned} x^4+3x^3+2x &= (4)^4+3(4)^3+2(4) \\ &= 256+3(64)+8 \\ &= 256+192+8 \\ &= 456 \end{aligned}$$

h. $$\begin{aligned} 3x^4+2x-3 &= 3(4)^4+2(4)-3 \\ &= 3(256)+8-3 \\ &= 768+8-3 \\ &= 773 \end{aligned}$$

2. a. $$\begin{aligned} p-2q &= 2-2(-3) \\ &= 2-(-6) \\ &= 2+6 \\ &= 8 \end{aligned}$$

b. $3q + r = 3(-3) + 6$
$= -9 + 6$
$= -3$

c. $5r + 3q = 5(6) + 3(-3)$
$= 30 + (-9)$
$= 21$

d. $3p - 2q - 5r = 3(2) - 2(-3) - 5(6)$
$= 6 - (-6) - 30$
$= 6 + 6 - 30$
$= -18$

e. $3r^2 - 5q = 3(6)^2 - 5(-3)$
$= 3(36) - (-15)$
$= 108 + 15$
$= 123$

f. $4p^2 + 3q^3 = 4(2)^2 + 3(-3)^3$
$= 4(4) + 3(-27)$
$= 16 + (-81)$
$= -65$

g. $q^2 - 3r^2 + 2p^3 = (-3)^2 - 3(6)^2 + 2(2)^3$
$= 9 - 3(36) + 2(8)$
$= 9 - 108 + 16$
$= -83$

h. $r^2 - 3q^2 + 4p^3 = (6)^2 - 3(-3)^2 + 4(2)^3$
$= 36 - 3(9) + 4(8)$
$= 36 - 27 + 32$
$= 41$

3. a. $f(1) = 3(1)^3 - 2(1) + 15$
$= 3(1) - 2 + 15$
$= 3 - 2 + 15$
$= 16$

b. $f(4) = 3(4)^3 - 2(4) + 15$
$= 3(64) - 8 + 15$
$= 192 - 8 + 15$
$= 199$

c. $f(-1) = 3(-1)^3 - 2(-1) + 15$
$= 3(-1) - (-2) + 15$
$= -3 + 2 + 15$
$= 14$

d. $$\begin{aligned} f(-10) &= 3(-10)^3 - 2(-10) + 15 \\ &= 3(-1000) - (-20) + 15 \\ &= -3000 + 20 + 15 \\ &= -2965 \end{aligned}$$

e. $$\begin{aligned} f(0) &= 3(0)^3 - 2(0) + 15 \\ &= 3(0) - 0 + 15 \\ &= 0 + 15 \\ &= 15 \end{aligned}$$

f. $$\begin{aligned} f\left(\frac{5}{2}\right) &= 3\left(\frac{5}{2}\right)^3 - 2\left(\frac{5}{2}\right) + 15 \\ &= 3\left(\frac{125}{8}\right) - 5 + 15 \\ &= \frac{375}{8} - 5 + 15 \\ &= \frac{375}{8} - \frac{40}{8} + \frac{120}{8} \\ &= \frac{455}{8} \text{ or } 56\frac{7}{8} \end{aligned}$$

g. $$\begin{aligned} f(-3) + f(2) &= \left[3(-3)^3 - 2(-3) + 15\right] + \left[3(2)^3 - 2(2) + 15\right] \\ &= \left[3(-27) - (-6) + 15\right] + \left[3(8) - 4 + 15\right] \\ &= \left[-81 + 6 + 15\right] + \left[24 - 4 + 15\right] \\ &= -60 + 35 \\ &= -25 \end{aligned}$$

h. $$\begin{aligned} f(-1) + 2f(0) &= \left[3(-1)^3 - 2(-1) + 15\right] + 2\left[3(0)^3 - 2(0) + 15\right] \\ &= \left[3(-1) - (-2) + 15\right] + 2\left[3(0) - 0 + 15\right] \\ &= \left[-3 + 2 + 15\right] + 2\left[0 + 15\right] \\ &= 14 + 30 \\ &= 44 \end{aligned}$$

4. The height above the water level is given by the equation $-4.9t^2 + 9.8t + 10$.
The time t is given in seconds.
How far is the diver above the water after 1 s, 2 s and 3 s?

a. After one second:

$$\begin{aligned} -4.9t^2 + 9.8t + 10 &= -4.9(1)^2 + 9.8(1) + 10 \\ &= -4.9 + 9.8 + 10 \\ &= 14.9 \end{aligned}$$

After one second, the diver is 14.9 m above the water.

b. After two seconds:

$$\begin{aligned}-4.9t^2+9.8t+10 &= -4.9(2)^2+9.8(2)+10\\ &= -4.9(4)+19.6+10\\ &= -19.6+19.6+10\\ &= 10\end{aligned}$$

After two seconds, the diver is 10 m above the water.

c. After three seconds:

$$\begin{aligned}-4.9t^2+9.8t+10 &= -4.9(3)^2+9.8(3)+10\\ &= -4.9(9)+29.4+10\\ &= -44.1+29.4+10\\ &= -4.7\end{aligned}$$

The diver has entered the water after three seconds.

5. A farmer may use a polynomial to decide how much fertilizer to purchase for next year. The polynomial will have variables for the amount of fertilizer per acre and the number of acres.

There are many other possible solutions.

6. a. $12d$ eggs

b. $4k$ tires

c. $60n$ minutes

d. $35p$ cents

e. $(45b+60c)$ cents

Extra Help

1. $2x-7$ when $x=3$

Enter	Display
2	2
$\boxed{\times}$	2
3	3
$\boxed{-}$	6
7	7
$\boxed{=}$	−1

2. $3x+1$ when $x=7$

Enter	Display
3	3
$\boxed{\times}$	3
7	7
$\boxed{+}$	21
1	1
$\boxed{=}$	22

3. $11-3x^3$ when $x=-2$

Enter	Display
11	11
$\boxed{-}$	11
3	3
$\boxed{\times}$	3
2	2
$\boxed{+/-}$	−2
$\boxed{y^x}$	−2
3	3
$\boxed{=}$	35

4. $3x^2-2x+3$ when $x=5$

Enter	Display
3	3
$\boxed{\times}$	3
5	5
$\boxed{x^2}$	25
$\boxed{-}$	75
2	2
$\boxed{\times}$	2
5	5
$\boxed{+}$	65
3	3
$\boxed{=}$	68

5. $-3x+2x^2-7$ when $x=-3$

Enter	Display
3	3
$\boxed{+/-}$	−3
$\boxed{\times}$	−3
3	3
$\boxed{+/-}$	−3
$\boxed{+}$	9
2	2
$\boxed{\times}$	2
3	3
$\boxed{+/-}$	−3
$\boxed{x^2}$	9
$\boxed{-}$	27
7	7
$\boxed{=}$	20

6. $2x^3+5x^2$ when $x=4$

Enter	Display
2	2
$\boxed{\times}$	2
4	4
$\boxed{x^y}$	4
3	3
$\boxed{+}$	128
5	5
$\boxed{\times}$	5
4	4
$\boxed{x^2}$	16
$\boxed{=}$	208

Extensions

1. Evaluate $2x^2 - 5x - 1$ when $x = 2.4$ (to the nearest tenth).

Enter	Display
2	2
$\boxed{\times}$	2
2.4	2.4
$\boxed{x^y}$	2.4
2	2
$\boxed{-}$	11.52
5	5
$\boxed{\times}$	2.4
$\boxed{-}$	−0.48
1	−1.48

−1.48 rounded to the nearest tenth is −1.5.

2. Evaluate $3x^2 - 7x + 3$ when $x = -1.7$ (to the nearest tenth).

Enter	Display
3	3
$\boxed{\times}$	3
1.7	1.7
$\boxed{+/-}$	−1.7
$\boxed{x^y}$	−1.7
2	2
$\boxed{-}$	8.67
7	7
$\boxed{\times}$	7
1.7	1.7
$\boxed{+/-}$	−1.7
$\boxed{+}$	20.57
3	3
$\boxed{=}$	23.57

23.57 rounded to the nearest tenth is 23.6.

3. Evaluate $(2x-3)^3 - x^4$ when $x = 2.3$ (to the nearest tenth).

Enter	Display
[	0
2	2
$\times$	2
2.3	2.3
$-$	4.6
3	3
]	1.6
x^y	1.6
3	3
$-$	4.096
2.3	2.3
x^y	2.3
4	4
=	-23.8881

-23.8881 rounded to the nearest tenth is -23.9.

4. Evaluate $3x^7 - 5x^6 - 10$ when $x = 0.8$ (to the nearest tenth).

Enter	Display
3	3
$\times$	3
0.8	0.8
x^y	0.8
7	7
$-$	0.6291456
5	5
$\times$	5
0.8	0.8
x^y	0.8
6	6
$-$	-0.6815744
10	10
=	-10.6815744

-10.6815744 rounded to the nearest tenth is -10.7.

Exploring Topic 3

Activity 1

Use the Multiplication or Product Law of exponents for powers with literal bases and whole number exponents.

1. a. $2^5 \times 2^{11} = 2^{5+11}$
   $= 2^{16}$

   b. $5^3 \times 5^{10} = 5^{3+10}$
   $= 5^{13}$

   c. $216^{32} \times 216^{16} = 216^{32+16}$
   $= 216^{48}$

   d. $138^{22} \times 138^{17} = 138^{22+17}$
   $= 138^{39}$

   e. $b^7 \times b^{21} = b^{7+21}$
   $= b^{28}$

   f. $y^{10} \times y^{12} = y^{10+12}$
   $= y^{22}$

   g. $m^{105} \times m^{236} = m^{105+236}$
   $= m^{341}$

   h. $n^{352} \times n^{102} = n^{352+102}$
   $= n^{454}$

   i. $5^8 \times 5^{12} \times 5^3 = 5^{8+12+3}$
   $= 5^{23}$

   j. $6^{12} \times 6^7 \times 6^3 = 6^{12+7+3}$
   $= 6^{22}$

   k. $(ab)^3 \times (ab)^{12} = (ab)^{3+12}$
   $= (ab)^{15}$

   l. $\left(mn^2\right)^3 \times \left(mn^2\right)^{11} = \left(mn^2\right)^{3+11}$
   $= \left(mn^2\right)^{14}$

2. The different factors being multiplied must have identical bases.

Activity 2

Use the Power of a Power Law for exponents.

1. a. $\left(2^3\right)^9 = 2^{3\times 9}$
$= 2^{27}$

b. $\left(3^5\right)^6 = 3^{5\times 6}$
$= 3^{30}$

c. $\left(125^5\right)^{30} = 125^{5\times 30}$
$= 125^{150}$

d. $\left(64^{15}\right)^{25} = 64^{15\times 25}$
$= 64^{375}$

e. $\left(x^{15}\right)^6 = {}^{15\times 6}$
$= x^{90}$

f. $\left(y^{22}\right)^8 = y^{22\times 8}$
$= y^{176}$

g. $\left(a^{125}\right)^{32} = a^{125\times 32}$
$= a^{4000}$

h. $\left(m^{32}\right)^{246} = m^{32\times 246}$
$= m^{7872}$

i. $\left[\left(5^2\right)^4\right]^7 = 5^{2\times 4\times 7}$
$= 5^{56}$

j. $\left[\left(3^3\right)^4\right]^{10} = 3^{3\times 4\times 10}$
$= 3^{120}$

k. $\left[(mn)^2\right]^{13} = (mn)^{2\times 13}$
$= (mn)^{26}$
$= m^{26}n^{26}$

l. $\left[(xy)^6\right]^7 = (xy)^{6\times 7}$
$= (xy)^{42}$
$= x^{42}y^{42}$

2. a. $\left(5^4\right)^2 \times \left(5^2\right)^6 = 5^{4\times 2} \times 5^{2\times 6}$
$= 5^8 \times 5^{12}$
$= 5^{8+12}$
$= 5^{20}$

b. $\left(3^5\right)^4 \times \left(3^6\right)^5 = 3^{5\times 4} \times 3^{6\times 5}$
$= 3^{20} \times 3^{30}$
$= 3^{20+30}$
$= 3^{50}$

c. $\left(m^2\right)^6 \times \left(m^3\right)^{12} = m^{2\times 6} \times m^{3\times 12}$
$= m^{12} \times m^{36}$
$= m^{12+36}$
$= m^{48}$

d. $\left(a^3\right)^6 \times \left(a^5\right)^5 = a^{3\times6} \times a^{5\times5}$
$= a^{18} \times a^{25}$
$= a^{18+25}$
$= a^{43}$

3. The Power of a Power Law is used when a power is raised to another exponent. The product of the two exponents is taken. This product is placed above the original base.

Activity 3

Use the Division or Quotient Law of exponents for powers with literal bases and whole number exponents.

1. a. $3^5 \div 3 = 3^{5-1}$
$= 3^4$

b. $4^9 \div 4^0 = 4^{9-0}$
$= 4^9$

c. $16^{482} \div 16^{107} = 16^{482-107}$
$= 16^{375}$

d. $23^{332} \div 23^{97} = 23^{332-97}$
$= 23^{235}$

e. $m^{23} \div m^9 = m^{23-9}$
$= m^{14}$

f. $n^{35} \div n^{15} = n^{35-15}$
$= n^{20}$

g. $x^{201} \div x^{157} = x^{201-157}$
$= x^{44}$

h. $y^{97} \div y^{63} = y^{97-63}$
$= y^{34}$

i. $3^{10} \div 3^4 \div 3^3 = 3^{10-4-3}$
$= 3^3$

j. $16^{15} \div 16^{10} \div 16^3 = 16^{15-10-3}$
$= 16^2$

k. $m^{23} \div \left(m^{24} \div m^{12}\right) = m^{23} \div m^{24-12}$
$= m^{23} \div m^{12}$
$= m^{23-12}$
$= m^{11}$

1. $$\begin{aligned} a^{17} \div \left(a^{20} \div a^{17}\right) &= a^{17} \div a^{20-17} \\ &= a^{17} \div a^{3} \\ &= a^{17-3} \\ &= a^{14} \end{aligned}$$

2. a. $$\begin{aligned} \left(3^{4}\right)^{4} \div \left(3^{2}\right)^{3} &= 3^{4\times4} \div 3^{2\times3} \\ &= 3^{16} \div 3^{6} \\ &= 3^{16-6} \\ &= 3^{10} \end{aligned}$$

b. $$\begin{aligned} \left(5^{4}\right)^{3} \div \left(5^{2}\right)^{5} &= 5^{4\times3} \div 5^{2\times5} \\ &= 5^{12} \div 5^{10} \\ &= 5^{12-10} \\ &= 5^{2} \end{aligned}$$

c. $$\begin{aligned} 16^{4} \times 16^{9} \div \left(16^{2}\right)^{5} &= 16^{4} \times 16^{9} \div 16^{2\times5} \\ &= 16^{4} \times 16^{9} \div 16^{10} \\ &= 16^{4+9-10} \\ &= 16^{3} \end{aligned}$$

d. $$\begin{aligned} 32^{5} \times \left(32^{11}\right) \div 32^{14} &= 32^{5} \times 32^{11\times2} \div 32^{14} \\ &= 32^{5} \times 32^{22} \div 32^{14} \\ &= 32^{5+22-14} \\ &= 32^{13} \end{aligned}$$

e. $$\begin{aligned} \left(m^{4} \times m^{3}\right)^{2} \div m^{12} &= \left(m^{4+3}\right)^{2} \div m^{12} \\ &= \left(m^{7}\right)^{2} \div m^{12} \\ &= m^{14} \div m^{12} \\ &= m^{14-12} \\ &= m^{2} \end{aligned}$$

f. $$\begin{aligned} m^{22} \times \left(m^{4} \div m\right)^{3} &= m^{22} \times \left(m^{4-1}\right)^{3} \\ &= m^{22} \times \left(m^{3}\right)^{3} \\ &= m^{22} \times m^{3\times3} \\ &= m^{22} \times m^{9} \\ &= m^{22+9} \\ &= m^{31} \end{aligned}$$

3. The dividend and the divisor must have identical bases. Then you may subtract the exponent of the divisor from the exponent of the dividend. The base remains the same.

Activity 4

Use the Power of a Product Law for exponents.

1. a. $(3a)^{11} = 3^{11}a^{11}$

   b. $(11b)^{9} = 11^{9}b^{9}$

   c. $(126x)^{503} = 126^{503}x^{503}$

   d. $(302y)^{126} = 302^{126}y^{126}$

   e. $(mn)^{102} = m^{102}n^{102}$

   f. $(rs)^{1026} = r^{1026}s^{1026}$

   g. $(14ab)^{32} = 14^{32}a^{32}b^{32}$

   h. $(15mn)^{16} = 15^{16}m^{16}n^{16}$

   i. $(abc)^{14} = a^{14}b^{14}c^{14}$

   j. $(xyz)^{22} = x^{22}y^{22}z^{22}$

   k. $(126rstu)^{20} = 126^{20}r^{20}s^{20}t^{20}u^{20}$

   l. $(196mnxyz)^{12} = 196^{12}m^{12}n^{12}x^{12}y^{12}z^{12}$

2. a. $\left(2m^{2}\right)^{4} = 2^{4}\left(m^{2}\right)^{4}$
   $= 2^{4}m^{2\times 4}$
   $= 2^{4}m^{8}$

   b. $\left(xy^{3}\right)^{10} = x^{10}\left(y^{3}\right)^{10}$
   $= x^{10}y^{3\times 10}$
   $= x^{10}y^{30}$

   c. $\left(2^{3}\times 2^{5}m^{3}\right)^{5} = \left(2^{3+5}m^{3}\right)^{5}$
   $= \left(2^{8}m^{3}\right)^{5}$
   $= \left(2^{8}\right)^{5}\left(m^{3}\right)^{5}$
   $= 2^{8\times 5}m^{3\times 5}$
   $= 2^{40}m^{15}$

d. $\left(5^{10} m^{4} \div 5^{8} m^{2}\right)^{3} = \left[\left(5^{10} \div 5^{8}\right)\left(m^{4} \div m^{2}\right)\right]^{3}$

$$= \left(5^{10-8} m^{4-2}\right)^{3}$$
$$= \left(5^{2} m^{2}\right)^{3}$$
$$= 5^{2\times3} m^{2\times3}$$
$$= 5^{6} m^{6}$$

e. $\left(xy^{2}\right)^{3}\left(x^{3}y\right)^{4} = (x)^{3}\left(y^{2}\right)^{3}\left(x^{3}\right)^{4}(y)^{4}$

$$= x^{3} y^{2\times3} x^{3\times4} y^{4}$$
$$= x^{3} y^{6} x^{12} y^{4}$$
$$= x^{3} x^{12} y^{6} y^{4}$$
$$= x^{3+12} y^{6+4}$$
$$= x^{15} y^{10}$$

f. $\left(10^{4} n^{3}\right)^{4} \div \left(10^{2} n^{2}\right)^{3} = \left(10^{4}\right)^{4}\left(n^{3}\right)^{4} \div \left[\left(10^{2}\right)^{3}\left(n^{2}\right)^{3}\right]$

$$= 10^{4\times4} n^{3\times4} \div \left(10^{2\times3} n^{2\times3}\right)$$
$$= 10^{16} n^{12} \div \left(10^{6} n^{6}\right)$$
$$= \left(10^{16} \div 10^{6}\right)\left(n^{12} \div n^{6}\right)$$
$$= 10^{16-6} n^{12-6}$$
$$= 10^{10} n^{6}$$

3. More than one factor must be inside the parentheses, and there must be an exponent outside the parentheses.

Activity 5

Use the Power of a Quotient Law for exponents.

1. a. $\left(\frac{5}{7}\right)^{11} = \frac{5^{11}}{7^{11}}$

   b. $\left(\frac{3}{11}\right)^{9} = \frac{3^{9}}{11^{9}}$

   c. $\left(\frac{123}{562}\right)^{93} = \frac{123^{93}}{562^{93}}$

   d. $\left(\frac{93}{256}\right)^{102} = \frac{93^{102}}{256^{102}}$

   e. $\left(\frac{5}{x}\right)^{26} = \frac{5^{26}}{x^{26}}$

   f. $\left(\frac{4}{3}\right)^{32} = \frac{4^{32}}{3^{32}}$

   g. $\left(\frac{m}{n}\right)^{15} = \frac{m^{15}}{n^{15}}$

   h. $\left(\frac{a}{b}\right)^{16} = \frac{a^{16}}{b^{16}}$

   i. $(5 \div q)^{11} = 5^{11} \div q^{11}$

   j. $(16 \div m)^{21} = 16^{21} \div m^{21}$

k. $$\left(\frac{\frac{m}{5}}{5}\right)^4 = \left(\frac{m}{5} \div 5\right)^4 = \left(\frac{m}{5} \times \frac{1}{5}\right)^4 = \left(\frac{m}{5^2}\right)^4 = \frac{m^4}{\left(5^2\right)^4} = \frac{m^4}{5^{2\times4}} = \frac{m^4}{5^8} \text{ or } \left(\frac{\frac{m^4}{5^4}}{5^4}\right)$$

l. $$\left(\frac{\frac{b}{c}}{c}\right)^5 = \left(\frac{b}{c} \div c\right)^5 = \left(\frac{b}{c} \times \frac{1}{c}\right)^5 = \left(\frac{b}{c^2}\right)^5 = \frac{b^5}{\left(c^2\right)^5} = \frac{b^5}{c^{2\times5}} = \frac{b^5}{c^{10}} \text{ or } \left(\frac{\frac{b^5}{c^5}}{c^5}\right)$$

Note: $$\left(\frac{\frac{b^5}{c^5}}{c^5}\right) = \frac{b^5}{c^5} \div c^5 = \frac{b^5}{c^5} \times \frac{1}{c^5} = \frac{b^5}{c^{5+5}} = \frac{b^5}{c^{10}}$$

2. a. $$\left(\frac{5}{m^2}\right)^6 = \frac{5^6}{\left(m^2\right)^6} = \frac{5^6}{m^{2\times6}} = \frac{5^6}{m^{12}}$$

b. $$\left(\frac{n^3}{5^2}\right)^4 = \frac{\left(n^3\right)^4}{\left(5^2\right)^4} = \frac{n^{3\times4}}{5^{2\times4}} = \frac{n^{12}}{5^8}$$

c. $$\left(\frac{2^4 m^2}{n^3}\right)^5 = \frac{\left(2^4\right)^5\left(m^2\right)^5}{\left(n^3\right)^5} = \frac{2^{4\times5} m^{2\times5}}{n^{3\times5}} = \frac{2^{20} m^{10}}{n^{15}}$$

d. $$\left(\frac{3^2 a}{b^3}\right)^4 = \frac{\left(3^2\right)^4 a^4}{\left(b^3\right)^4}$$

$$= \frac{3^{2\times4} a^4}{b^{3\times4}}$$

$$= \frac{3^8 a^4}{b^{12}}$$

e. $$\left(\frac{m^2}{n^3}\right)^4 \left(\frac{n^2}{m^5}\right)^3 = \frac{\left(m^2\right)^4}{\left(n^3\right)^4} \times \frac{\left(n^2\right)^3}{\left(m^5\right)^3}$$

$$= \frac{m^{2\times4}}{n^{3\times4}} \times \frac{n^{2\times3}}{m^{5\times3}}$$

$$= \frac{m^8}{n^{12}} \times \frac{n^6}{m^{15}}$$

$$= \frac{m^8}{m^{15}} \times \frac{n^6}{n^{12}}$$

$$= m^{8-15} \times n^{6-12}$$

$$= m^{-7} n^{-6} \text{ or } \frac{1}{m^7 n^6}$$

f. $$\left(\frac{x^3}{y}\right)^5 \div \left(\frac{x^2}{y^4}\right)^3 = \frac{\left(x^3\right)^5}{y^5} \div \frac{\left(x^2\right)^3}{\left(y^4\right)^3}$$

$$= \frac{x^{3\times5}}{y^5} \div \frac{x^{2\times3}}{y^{4\times3}}$$

$$= \frac{x^{15}}{y^5} \div \frac{x^6}{y^{12}}$$

$$= \frac{x^{15}}{y^5} \times \frac{y^{12}}{x^6}$$

$$= \frac{x^{15}}{x^6} \times \frac{y^{12}}{y^5}$$

$$= x^{15-6} y^{12-5}$$

$$= x^9 y^7$$

3. The Power of a Quotient Law is used when the dividend and the divisor are inside a pair of parentheses and there is an exponent after the parentheses. The exponent outside the parentheses becomes the exponent of both the dividend and the divisor in the simplified form.

Activity 6

Verify each of the power laws for exponents.

1. a.

LS	RS
$2^3 \times 2^1$	2^4
$2\times2\times2\times2$	2^4
2^4	2^4

LS = RS

b.

LS	RS
$\left(\frac{x}{y}\right)^3$	$\frac{x^3}{y^3}$
$\frac{x}{y}\times\frac{x}{y}\times\frac{x}{y}$	$\frac{x^3}{y^3}$
$\frac{x\times x\times x}{y\times y\times y}$	$\frac{x^3}{y^3}$
$\frac{x^3}{y^3}$	$\frac{x^3}{y^3}$

LS = RS

c.

LS	RS
$\left(x^3\right)^3$	x^9
$x^3 \times x^3 \times x^3$	x^9
$(x\times x\times x)(x\times x\times x)(x\times x\times x)$	x^9
x^9	x^9

LS = RS

d.

LS	RS
$(3n)^5$	$3^5 n^5$
$3n\times3n\times3n\times3n\times3n$	$3^5 n^5$
$(3\times3\times3\times3\times3)(n\times n\times n\times n\times n)$	$3^5 n^5$
$3^5 n^5$	$3^5 n^5$

LS = RS

e.

LS	RS
$a^{12} \div a^4$	a^8
$\frac{a\times a\times a\times a\times \overset{1}{\cancel{a}}\times \overset{1}{\cancel{a}}\times \overset{1}{\cancel{a}}\times \overset{1}{\cancel{a}}\times a\times a\times a\times a}{\underset{1}{\cancel{a}}\times \underset{1}{\cancel{a}}\times \underset{1}{\cancel{a}}\times \underset{1}{\cancel{a}}}$	a^8
$a\times a\times a\times a\times a\times a\times a\times a$	a^8
a^8	a^8

LS = RS

f.

LS	RS
$4^9 \div 4^5$	4^4
$\frac{4\times4\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times4\times4}{\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}}$	4^4
$4\times4\times4\times4$	4^4
4^4	4^4

LS = RS

2. Show that $\dfrac{a^m \bullet a^n}{a^x} = a^{(m+n-x)}$ by substituting the following values.

Method 1: where $m = 2,\ n = 3,\ x = 4$

$$\frac{a^2 \bullet a^3}{a^4} = a^{(2+3-4)}$$
$$= a^1$$

LS	RS
$\dfrac{a^2 \bullet a^3}{a^4}$	$a^{(2+3-4)}$
$\dfrac{a \bullet a \bullet a \bullet a \bullet a}{a \bullet a \bullet a \bullet a}$	a^{5-4}
$\dfrac{\overset{1}{\cancel{a}} \bullet \overset{1}{\cancel{a}} \bullet \overset{1}{\cancel{a}} \bullet \overset{1}{\cancel{a}} \bullet a}{\underset{1}{\cancel{a}} \bullet \underset{1}{\cancel{a}} \bullet \underset{1}{\cancel{a}} \bullet \underset{1}{\cancel{a}}}$	a^1
a	a

(Cancel the common factors.)

LS = RS

Method 2: where $a = 5,\ m = 2,\ n = 3,\ x = 4$

$$\frac{5^2 \bullet 5^3}{5^4} = 5^{(2+3-4)}$$
$$= 5^1$$

LS	RS
$\dfrac{5^2 \bullet 5^3}{5^4}$	$5^{(2+3-4)}$
$\dfrac{\overset{1}{\cancel{5}} \bullet \overset{1}{\cancel{5}} \bullet \overset{1}{\cancel{5}} \bullet \overset{1}{\cancel{5}} \bullet 5}{\underset{1}{\cancel{5}} \bullet \underset{1}{\cancel{5}} \bullet \underset{1}{\cancel{5}} \bullet \underset{1}{\cancel{5}}}$	5^{5-4}
5	5^1
5	5

LS = RS

Extra Help

1. a. $x^5 \times x^7 = x^{5+7}$
$= x^{12}$

b. $a^4 \times a^{11} = a^{4+11}$
$= a^{15}$

c. $2^4 \times 2^{19} = 2^{4+19}$
$= 2^{23}$

d. $y^{14} \times y^{26} = y^{14+26}$
$= y^{40}$

2. a. $\frac{x^{14}}{x^{9}} = x^{14-9}$

$= x^{5}$

b. $\frac{y^{24}}{y^{16}} = y^{24-16}$

$= y^{8}$

c. $\frac{5^{7}}{5^{3}} = 5^{7-3}$

$= 5^{4}$

d. $\frac{k^{100}}{k^{25}} = k^{100-75}$

$= k^{75}$

e. $\frac{z^{17}}{z^{14}} = z^{17-14}$

$= z^{3}$

3. a. $\left(2^{5}\right)^{2} = 2^{5\times 2}$

$= 2^{10}$

b. $\left(x^{4}\right)^{7} = x^{4\times 7}$

$= x^{28}$

c. $\left(y^{3}\right)^{2} = y^{3\times 2}$

$= y^{6}$

d. $\left(z^{5}\right)^{7} = z^{5\times 7}$

$= z^{35}$

e. $\left(k^{9}\right)^{4} = k^{9\times 4}$

$= k^{36}$

4. a. $\left(3y^{2}\right)^{4} = 3^{4}\left(y^{2}\right)^{4}$

$= 3^{4}y^{8}$

b. $\left(x^{3}y^{5}\right)^{2} = \left(x^{3}\right)^{2}\left(y^{5}\right)^{2}$

$= x^{6}y^{10}$

c. $\left(k^{4}z^{5}\right)^{2} = \left(k^{4}\right)^{2}\left(z^{5}\right)^{2}$

$= k^{8}z^{10}$

d. $\left(2k^{3}z^{2}\right)^{3} = 2^{3}\left(k^{3}\right)^{3}\left(z^{2}\right)^{3}$

$= 2^{3}k^{9}z^{6}$

$= 8k^{9}z^{6}$

5. a. $\left(\frac{x}{y}\right)^{2} = \frac{x^{2}}{y^{2}}$

b. $\left(\frac{x^{2}}{y^{3}}\right)^{2} = \frac{\left(x^{2}\right)^{2}}{\left(y^{3}\right)^{2}}$

$= \frac{x^{4}}{y^{6}}$

c. $\left(\frac{5^{3}}{3^{4}}\right)^{2} = \frac{\left(5^{3}\right)^{2}}{\left(3^{4}\right)^{2}}$

$= \frac{5^{6}}{3^{8}}$

d. $\left(\frac{x^{3}}{k^{2}}\right)^{4} = \frac{\left(x^{3}\right)^{4}}{\left(k^{2}\right)^{4}}$

$= \frac{x^{12}}{k^{8}}$

Extensions

If you examine the money received and the day in which it was received, you should see a pattern developing. This pattern is in the form of a power.

Aug 1	1¢	2^{0}
Aug 2	2¢	2^{1}
Aug 3	4¢	2^{2}

Aug 4	8¢	2^3
Aug 5	16¢	2^4
Aug 6	32¢	2^5
Aug 7	64¢	2^6

The exponent in the pattern is always one less than the day in which he received the money.

On Aug 7 he received 2^6 cents.
On Aug 8 he received 2^7 cents.
On Aug 9 he received 2^8 cents.
⋮
∴ On Aug 31 he received 2^{30} cents.

Now evaluate 2^{30} with your calculator.

Enter	Display
2	2
$\boxed{x^y}$	2
30	30
$\boxed{=}$	1073741824

Note: Jacob would also have the money he received on each day prior to August 31 which would include (2^{29})¢, (2^{28})¢, (2^{27})¢ and so on to (2^0)¢. Use your calculator to find the entire amount. Are you surprised to get such a large amount?

On August 31 Jacob would receive 1 073 741 824 cents or $10 737 418.24.

Jacob would choose the second alternative.

Exploring Topic 4

Activity 1

Use zero exponents.

1. a. $5^0 = 1$
   b. $12^0 = 1$
   c. $256^0 = 1$
   d. $192^0 = 1$
   e. $x^0 = 1$
   f. $d^0 = 1$
   g. $(2m)^0 = 1$
   h. $(xy)^0 = 1$
   i. $\left(\frac{16x}{y}\right)^0 = 1$
   j. $\left(\frac{m^2}{7n}\right)^0 = 1$
   k. $2m^0 = 2\times1$
      $= 2$
   l. $\frac{x^0}{y} = \frac{1}{y}$

m. $\left[\left(2m^2\right)^0\right]^2 = (1)^2$

$= 1$

n. $2m^0 \times 15nm^2 = 2 \times 1 \times 15nm^2$

$= 30nm^2$

2. There is no number you can multiply zero by to give a product of 1.

Activity 2

Use negative exponents.

1. a. $2^{-8} = \dfrac{1}{2^8}$

b. $15^{-20} = \dfrac{1}{15^{20}}$

c. $293^{-92} = \dfrac{1}{293^{92}}$

d. $302^{-1024} = \dfrac{1}{302^{1024}}$

e. $a^{-13} = \dfrac{1}{a^{13}}$

f. $m^{-26} = \dfrac{1}{m^{26}}$

g. $g^{-249} = \dfrac{1}{g^{249}}$

h. $n^{-1029} = \dfrac{1}{n^{1029}}$

i. $(2m)^{-14} = \dfrac{1}{(2m)^{14}}$

$= \dfrac{1}{2^{14} m^{14}}$

j. $\left(\dfrac{5}{y}\right)^{-3} = \left(\dfrac{y}{5}\right)^3$

$= \dfrac{y^3}{5^3}$

k. $\left(x^{-3}y^2\right)^3\left(x^2y^{-3}\right)^2 = \left(x^{-9}y^6\right)\left(x^4y^{-6}\right)$

$= \left(x^{-9}x^4\right)\left(y^6y^{-6}\right)$

$= x^{-5}y^0$

$= \dfrac{1}{x^5}$

l. $$\frac{a^{-2}b}{a^{-4}b^{-3}} = a^{-2-(-4)}b^{1-(-3)}$$
$$= a^{-2+4}b^{1+3}$$
$$= a^2b^4$$

m. $$\left(2m^2n^2\right)^{-1} \div \left(2m^2y\right)^{-2}$$
$$= (2)^{-1}\left(m^2\right)^{-1} \div (2)^{-2}\left(m^2\right)^{-2}(y)^{-2}$$
$$= 2^{-1}m^{-2}n^{-2} \div 2^{-2}m^{-4}y^{-2}$$
$$= \frac{2^{-1}m^{-2}n^{-2}}{2^{-2}m^{-4}y^{-2}}$$
$$= \frac{2^{-1-(-2)}m^{-2-(-4)}n^{-2}}{y^{-2}}$$
$$= \frac{2^{-1+2}m^{-2+4}n^{-2}}{y^{-2}}$$
$$= \frac{2m^2n^{-2}}{y^{-2}}$$
$$= \frac{2m^2y^2}{n^2}$$

n. $$\left(\frac{x^{-2}y}{z^3}\right)^{-3} \div \left(\frac{x^5y^{-2}}{z^3}\right)^{-1} = \frac{\left(x^{-2}\right)^{-3}(y)^{-3}}{\left(z^3\right)^{-3}} \div \frac{\left(x^5\right)^{-1}\left(y^{-2}\right)^{-1}}{\left(z^3\right)^{-1}}$$
$$= \frac{x^{-2\times(-3)}y^{-3}}{z^{3\times(-3)}} \div \frac{x^{5\times(-1)}y^{-2\times(-1)}}{z^{3\times(-1)}}$$
$$= \frac{x^6y^{-3}}{z^{-9}} \div \frac{x^{-5}y^2}{z^{-3}}$$
$$= \frac{x^6z^9}{y^3} \div \frac{y^2z^3}{x^5}$$
$$= \frac{x^6z^9}{y^3} \times \frac{x^5}{y^2z^3}$$
$$= \frac{x^{6+5}z^{9-3}}{y^{3+2}}$$
$$= \frac{x^{11}z^6}{y^5}$$

2. A negative exponent means to take the reciprocal of the number, and a negative base refers to the sign of the number.

Extra Help

1. 5^{-2}

Enter	Display
5	5
x^y	5
2	2
+/–	−2
=	0.04

2. $(-5)^2$

Enter	Display
5	5
+/–	−5
x^y	−5
2	2
=	25

Note: If your calculator displays an error message (E) when you calculate $\frac{1}{(2)^3}$, change the power to 2^{-3}, in which case the display would be as follows.

Enter	Display
2	2
x^y	2
3	3
+/–	−3
=	0.125

3. $(-5)^{-2}$

Enter	Display
5	5
+/–	−5
x^y	−5
2	2
+/–	−2
=	0.04

4. $\frac{1}{(2)^3}$

Enter	Display
1	1
÷	1
2	2
x^y	2
3	3
=	0.125

5. $\frac{1}{(-2)^3}$

Enter	Display
1	1
÷	1
2	2
+/–	−2
x^y	−2
3	3
=	−0.125

6. $\frac{1}{(-2)^{-3}}$

Enter	Display
1	1
÷	1
2	2
+/–	−2
x^y	−2
3	3
+/–	−3
=	−8

Extensions

1. $(1.7)^{-3} \times (4.2)^2$

Enter	Display
1.7	1.7
$\boxed{x^y}$	1.7
3	3
$\boxed{+/-}$	−3
$\boxed{\times}$	0.203541624
4.2	4.2
$\boxed{x^y}$	4.2
2	2
$\boxed{=}$	3.590474252

$\therefore (1.7)^{-3} \times (4.2)^2 \doteq 3.6$

2. $(2.4)^{-1.7} \times (4.3)^{2.6}$

Enter	Display
2.4	2.4
$\boxed{x^y}$	2.4
1.7	1.7
$\boxed{+/-}$	−1.7
$\boxed{\times}$	0.225756825
4.3	4.3
$\boxed{x^y}$	4.3
2.6	2.6
$\boxed{=}$	10.01518264

$\therefore (2.4)^{-1.7} \times (4.3)^{2.6} \doteq 10.0$

3. $\dfrac{2.4}{(5.2)^{-0.9}} \times (1.5)^{-3}$

Enter	Display
2.4	2.4
$\boxed{\div}$	2.4
5.2	5.2
$\boxed{x^y}$	5.2
0.9	0.9
$\boxed{+/-}$	−0.9
$\boxed{\times}$	10.58313293
1.5	1.5
$\boxed{x^y}$	1.5
3	3
$\boxed{+/-}$	−3
$\boxed{=}$	3.135743089

$\therefore \dfrac{2.4}{(5.2)^{-0.9}} \times (1.5)^{-3} \doteq 3.1$

4. $(1.7)^{2.3} \times (2.4)^{3.1} \times (1.1)^{-7.1}$

Enter	Display
1.7	1.7
$\boxed{x^y}$	1.7
2.3	2.3
$\boxed{\times}$	3.388695291
2.4	2.4
$\boxed{x^y}$	2.4
3.1	3.1
$\boxed{\times}$	51.13136284
1.1	1.1
$\boxed{x^y}$	1.1
7.1	7.1
$\boxed{+/-}$	−7.1
$\boxed{=}$	25.98958255

$\therefore (1.7)^{2.3} \times (2.4)^{3.1} \times (1.1)^{-7.1} \doteq 26.0$

5. a. 6.37×10^{6} m

b. 1.60×10^{-19} J

c. 3×10^{8} m/s

d. 9.81×10^{0} m/s^{2}

e. 3×10^{11} m

6. a. 356 000 kg

b. 0.000 000 000 000 000 000 000 000 001 67 kg

c. 11 000 000/m

d. 1

7. a.
$$\begin{aligned}590\,000\times120\,400&=\left(5.9\times10^{5}\right)\times\left(1.204\times10^{5}\right)\\&=(5.9\times1.204)\times\left(10^{5}\times10^{5}\right)\\&=(7.1036)\times\left(10^{5+5}\right)\\&=7.1036\times10^{10}\end{aligned}$$

b.
$$\begin{aligned}0.000\,001\,03\times0.000\,34&=\left(1.03\times10^{-6}\right)\times\left(3.4\times10^{-4}\right)\\&=(1.03\times3.4)\times\left(10^{-6}\times10^{-4}\right)\\&=(3.502)\times\left(10^{-6+-4}\right)\\&=3.502\times10^{-10}\end{aligned}$$

c.
$$\begin{aligned}0.000\,102\times3\,000\,000&=\left(1.02\times10^{-4}\right)\times\left(3\times10^{6}\right)\\&=(1.02\times3)\times\left(10^{-4}\times10^{6}\right)\\&=(3.06)\times\left(10^{-4+6}\right)\\&=3.06\times10^{2}\end{aligned}$$

d.
$$\begin{aligned}340\,000\div0.000\,068&=\left(3.4\times10^{5}\right)\div\left(6.8\times10^{-5}\right)\\&=\frac{3.4}{6.8}\times\frac{10^{5}}{10^{-5}}\\&=0.5\times10^{5-(-5)}\\&=0.5\times10^{10}\quad\text{(not in scientific notation)}\\&=\left(5.0\times10^{-1}\right)\times\left(10^{10}\right)\\&=5.0\times10^{-1+10}\\&=5.0\times10^{9}\end{aligned}$$

e.
$$\begin{aligned}0.000\,098\div24\,500\,000&=\left(9.8\times10^{-5}\right)\div\left(2.45\times10^{7}\right)\\&=\frac{9.8}{2.45}\times\frac{10^{-5}}{10^{7}}\\&=(4.0)\times\left(10^{-5-7}\right)\\&=4.0\times10^{-12}\end{aligned}$$

8. a.

$$\begin{aligned}\frac{275\,000\times 67\,000}{285\,000} &= \frac{\left(2.75\times 10^{5}\right)\times\left(6.7\times 10^{4}\right)}{2.85\times 10^{5}}\\ &= \left(\frac{2.75\times 6.7}{2.85}\right)\times\left(\frac{10^{5}\times 10^{4}}{10^{5}}\right)\\ &= (6.5)\times\left(10^{5+4-5}\right)\\ &= 6.5\times 10^{4}\end{aligned}$$

b.

$$\begin{aligned}\frac{189\,000\times 0.004}{149\,000} &= \frac{\left(1.89\times 10^{5}\right)\times\left(4\times 10^{-3}\right)}{1.49\times 10^{5}}\\ &= \left(\frac{1.89\times 4}{1.49}\right)\times\left(\frac{10^{5}\times 10^{-3}}{10^{5}}\right)\\ &= (5.1)\times\left(10^{5+(-3)-5}\right)\\ &= 5.1\times 10^{-3}\end{aligned}$$

c.

$$\begin{aligned}\frac{78\,500}{34\,000\times 0.004\,82} &= \frac{7.85\times 10^{4}}{\left(3.4\times 10^{4}\right)\times\left(4.82\times 10^{-3}\right)}\\ &= \left(\frac{7.85}{3.4\times 4.82}\right)\times\left(\frac{10^{4}}{10^{4}\times 10^{-3}}\right)\\ &= (0.48)\times\left[10^{4-4-(-3)}\right]\\ &= \left(4.8\times 10^{-1}\right)\times\left(10^{3}\right)\\ &= 4.8\times 10^{-1+3}\\ &= 4.8\times 10^{2}\end{aligned}$$

d.

$$\begin{aligned}\frac{0.000\,004\,81}{0.000\,241\times 340\,000} &= \frac{4.81\times 10^{-6}}{\left(2.41\times 10^{-4}\right)\times\left(3.4\times 10^{5}\right)}\\ &= \left(\frac{4.81}{2.41\times 3.4}\right)\times\left(\frac{10^{-6}}{10^{-4}\times 10^{5}}\right)\\ &= (0.59)\times\left[10^{-6-(-4)-5}\right]\\ &= \left(5.9\times 10^{-1}\right)\times\left(10^{-7}\right)\\ &= 5.9\times 10^{-1+(-7)}\\ &= 5.9\times 10^{-8}\end{aligned}$$

e. $$\frac{1}{78\,000 \times 0.000\,0012} = \frac{1 \times 10^0}{(7.8 \times 10^4) \times (1.2 \times 10^{-6})}$$

$$= \left(\frac{1}{7.8 \times 1.2}\right) \times \left(\frac{10^0}{10^4 \times 10^{-6}}\right)$$

$$= (0.11) \times \left[10^{0-4-(-6)}\right]$$

$$= (1.1 \times 10^{-1}) \times 10^2$$

$$= 1.1 \times 10^{-1+2}$$

$$= 1.1 \times 10^1 \text{ or } 1.1 \times 10$$

9. a. $$\frac{(7.25 \times 10^{12}) \times (4.05 \times 10^5)}{3.098 \times 10^{-16}} = \left(\frac{7.25 \times 4.05}{3.098}\right) \times \left(\frac{10^{12} \times 10^5}{10^{-16}}\right)$$

$$= 9.5 \times \left[10^{12+5-(-16)}\right]$$

$$= 9.5 \times 10^{33}$$

b. $$\frac{(3.4 \times 10^{-3}) \times (9.00 \times 10^{-12})}{4.009 \times 10^{-8}} = \left(\frac{3.4 \times 9.00}{4.009}\right) \times \left(\frac{10^{-3} \times 10^{-12}}{10^{-8}}\right)$$

$$= (7.6) \times \left[10^{-3+(-12)-(-8)}\right]$$

$$= 7.6 \times 10^{-7}$$

c. $$\frac{(1.0 \times 10^4) \times (3 \times 10^{34})}{2.3 \times 10^{20}} = \left(\frac{1.0 \times 3}{2.3}\right) \times \left(\frac{10^4 \times 10^{34}}{10^{20}}\right)$$

$$= (1.3) \times (10^{4+34-20})$$

$$= 1.3 \times 10^{18}$$

d. $$\frac{(2.3 \times 10^{109}) \times (4.5 \times 10^{209})}{4.098 \times 10^{99}} = \left(\frac{2.3 \times 4.5}{4.098}\right) \times \left(\frac{10^{109} \times 10^{209}}{10^{99}}\right)$$

$$= (2.5) \times (10^{109+209-99})$$

$$= 2.5 \times 10^{219}$$

10. a. Six hours per day are spent in a classroom.
200 days per year are spent in a classroom.
Twelve years are spent in a classroom.
There are 3600 seconds in an hour.
How many seconds are spent in the classroom?

$$\frac{6 \text{ h}}{\text{d}} \times \frac{200 \text{ d}}{\text{a}} \times 12 \text{ a} \times \frac{3600 \text{ s}}{\text{h}} = 6 \times 200 \times 12 \times 3600 \text{ s}$$

$$= 51\,840\,000 \text{ s}$$

$$= 5.184 \times 10^7 \text{ s}$$

You would spend 5.184×10^7 s in a classroom in twelve years.

b. A page has a thickness of 9.0×10^{-3} cm.
A book is 7 cm thick.
How many pages are in the book?

$$\frac{\text{pg}}{9.0\times10^{-3}\text{ cm}}\times7\text{ cm}=\frac{7\text{ pg}}{9.0\times10^{-3}}$$
$$=\frac{7}{9.0}\times10^{3}$$
$$=0.8\times10^{3}$$
$$=8\times10^{-1}\times10^{3}$$
$$=8\times10^{-1+3}$$
$$=8\times10^{2}$$

There are 8×10^{2} pages in a book that is 7 cm thick.

c. The Andromeda galaxy is 2.2×10^{6} light years away.
A light year is the distance light travels in one year.
The speed of light is 3.0×10^{8} m/s.

Remember there are
$\frac{365\text{ d}}{\text{year}}\times\frac{24\text{ h}}{\text{d}}\times\frac{60\text{ min}}{\text{h}}\times\frac{60\text{ s}}{\text{min}}=3.1536\times10^{7}$ s in one year.

$$2.2\times10^{6}\text{ a}\times\frac{3.0\times10^{8}\text{ m}}{\text{s}}\times\frac{3.1536\times10^{7}\text{ s}}{\text{a}}$$
$$=\left(2.2\times10^{6}\right)\times\left(3.0\times10^{8}\text{ m}\right)\times\left(3.1536\times10^{7}\right)$$
$$=(2.2\times3.0\times3.1536)\times\left(10^{6}\times10^{8}\times10^{7}\right)\text{ m}$$
$$=(21)\times\left(10^{6+8+7}\right)\text{ m}$$
$$=21\times10^{21}\text{ m}$$
$$=2.1\times10^{1+21}\text{ m}$$
$$=2.1\times10^{22}\text{ m}$$

The Andromeda Galaxy is 2.1×10^{22} m away from the Earth. Light from the Adromeda Galaxy will travel 2.1×10^{22} m to Earth.

d. 6.02×10^{23} atoms have a mass of 16 g. What is the mass of 1 atom?

$$\frac{\text{number}}{\text{mass}}=\frac{\text{number}}{\text{mass}}$$

$$\frac{6.02\times10^{23}\ \text{atoms}}{16\ \text{g}}=\frac{1\ \text{atoms}}{m}$$

$$m=\frac{16\ \text{g}\times1\ \text{atom}}{6.02\times10^{23}\ \text{atoms}}$$

$$=\left(\frac{16}{6.02}\right)\times10^{-23}\ \text{g}$$

$$=(2.7)\times10^{-23}\ \text{g}$$

$$=2.7\times10^{-23}\ \text{g}$$

One atom of oxygen will have a mass of 2.7×10^{-23} g.

11. a. $9\,350\,000\,000\times1\,350\,000\,000=(9.35\times10^{9})\times(1.35\times10^{9})$

Enter	Display
9.35	9.35
EXP	9.35 00
9	9.35 09
×	9350000000 (or 9.35 09)
1.35	1.35
EXP	1.35 00
9	1.35 09
=	1.26225 19

$\therefore 9\,350\,000\,000\times1\,350\,000\,000 \doteq 1.26\times10^{19}$

b. $0.000\,000\,0451\times0.000\,000\,000\,009\,28$

$=(4.51\times10^{-8})\times(9.28\times10^{-12})$

Enter	Display
4.51	4.51
EXP	4.51 00
8	4.51 08
+/–	4.51 –08
×	0.000000045 (or 4.51 –08)
9.28	9.28
EXP	9.28 00
12	9.28 12
+/–	9.28 –12
=	4.18528 –19

$\therefore 0.000\,000\,0451\times0.000\,000\,000\,009\,28 \doteq 4.19\times10^{-19}$

c. 8 550 000 000 000 ÷ 2 850 000 000

$$=(8.55\times10^{12})\div(2.85\times10^{9})$$

Enter	Display
8.55	8.55
$\boxed{\text{EXP}}$	8.55 00
12	8.55 12
$\boxed{\div}$	8.55 12
2.85	2.85
$\boxed{\text{EXP}}$	2.85 00
9	2.85 09
$\boxed{=}$	3000

$\therefore 8\,550\,000\,000\,000\div2\,850\,000\,000=3000$ or 3.00×10^{3}

d. 0.000 000 000 009 513 ÷ 3 171 000 000

$$=(9.513\times10^{-12})\div(3.171\times10^{9})$$

Enter	Display
9.513	9.513
$\boxed{\text{EXP}}$	9.513 00
12	9.513 12
$\boxed{+/-}$	9.513 −12
$\boxed{\div}$	9.513 −12
3.171	3.171
$\boxed{\text{EXP}}$	3.171 00
9	3.171 09
$\boxed{=}$	3 −21

$\therefore 0.000\,000\,000\,009\,513\div3\,171\,000\,000=3.00\times10^{-21}$

e. $\left(7.29\times10^{21}\right)\times\left(4.21\times10^{15}\right)$

Enter	Display
7.29	7.29
$\boxed{\text{EXP}}$	7.29 00
21	7.29 21
$\boxed{\times}$	7.29 21
4.21	4.21
$\boxed{\text{EXP}}$	4.21 00
15	4.21 15
$\boxed{=}$	3.06909 37

$\therefore\left(7.29\times10^{21}\right)\times\left(4.21\times10^{15}\right)\doteq3.07\times10^{37}$

f. $\left(3.82\times10^{15}\right)\times\left(4.008\times10^{-22}\right)$

Enter	Display
3.82	3.82
$\boxed{\text{EXP}}$	3.82 00
15	3.82 15
$\boxed{\times}$	3.82 15
4.008	4.008
$\boxed{\text{EXP}}$	4.008 00
22	4.008 22
$\boxed{+/-}$	4.008 −22
$\boxed{=}$	1.531056 −06

$\therefore\left(3.82\times10^{15}\right)\times\left(4.008\times10^{-22}\right)\doteq1.53\times10^{-6}$

g. $\left(9.28\times10^{42}\right)\div\left(1.856\times10^{19}\right)$

Enter	Display
9.28	9.28
EXP	9.28 00
42	9.28 42
÷	9.28 42
1.856	1.856
EXP	1.856 00
19	1.856 19
=	5. 23

$\therefore\left(9.28\times10^{42}\right)\div\left(1.856\times10^{19}\right)=5.00\times10^{23}$

h. $\left(1.0092\times10^{-18}\right)\div\left(2.0184\times10^{-12}\right)$

Enter	Display
1.0092	1.0092
EXP	1.0092 00
18	1.0092 18
+/–	1.0092 –18
÷	1.0092 –18
2.0184	2.0184
EXP	2.0184 00
12	2.0184 12
+/–	2.0184 –12
=	5. –07

$\therefore\left(1.009\times10^{-18}\right)\div\left(2.0184\times10^{-12}\right)=5.00\times10^{-7}$

i. $\dfrac{9\,870\,000\,000 \times 1\,025\,000\,000}{9\,085\,000\,000\,000} = \dfrac{(9.87 \times 10^{9}) \times (1.025 \times 10^{9})}{9.085 \times 10^{12}}$

Enter	Display
9.87	9.87
EXP	9.87 00
9	9.87 09
×	9870000000 (or 9.87 09)
1.025	1.025
EXP	1.025 00
9	1.025 09
÷	1.011675 19
9.085	9.085
EXP	9.085 00
12	9.085 12
=	1113566.318

$\therefore \dfrac{9\,870\,000\,000 \times 1\,025\,000\,000}{9\,085\,000\,000\,000} \doteq 1.11 \times 10^{6}$

j. $\dfrac{598\,200\,000\,000}{7\,003\,900\,000 \times 10\,940\,000\,000}$

$= \dfrac{5.982 \times 10^{11}}{(7.0039 \times 10^{9}) \times (1.094 \times 10^{10})}$

Enter	Display
5.982	5.982
EXP	5.982 00
11	5.982 11
÷	5.982
(	(01 0
7.0039	7.0039
EXP	7.0039 00
9	7.0039 09
×	7.003 900 000
1.094	1.094
EXP	1.094 00
10	1.094 10
)	7.6622666 19
=	7.807089354 −09

$$\therefore \frac{598\,200\,000\,000}{7\,003\,900\,000 \times 10\,940\,000\,000} \doteq 7.81 \times 10^{-9}$$

Note: The displays in this question will vary in several lines for different calculators.

12. a. In one year there are

$(365 \times 24 \times 60 \times 60)$ seconds $= 3.1536 \times 10^{7}$ seconds.

In one year, light will travel

$\left(3.0 \times 10^{5}\right) \times \left(3.1536 \times 10^{7}\right) = 9.4608 \times 10^{12}$ km.

In 2.6×10^{9} years, light will travel a distance of

$\left(2.6 \times 10^{9}\right) \times \left(9.460\,8 \times 10^{12}\right) = 2.459\,808 \times 10^{22}$ km.

The distance from earth is about 2.5×10^{22} km.

b. The gold's value was $1.87 \times 10^{7} \times 546 = 1.021\,02 \times 10^{10}$.
The gold's value was about 10.2 billion dollars.

c.

$$v = f\lambda$$

$$3 \times 10^{8} = f \times \left(2.3 \times 10^{-12}\right)$$

$$\frac{3 \times 10^{8}}{2.3 \times 10^{-12}} = f$$

$$1.304\,348 \times 10^{20} = f$$

The frequency is about 1.3×10^{20} units.

13.

Number	Significant (yes or no)	Rule
1.076	**Yes**	**2a**
3.0704	**Yes**	**2a**
40.800	**Yes**	**2a**
0.005 03	**No**	**2d**
6.00	**Yes**	**2c**
0.69	**No**	**2d**

14. a. 3.5607 g
The zero is significant.
There are five significant digits in all.

b. 6.05×10^{5} mL
The zero is significant.
There are three significant digits in all.

c. 620.8 L
The zero is significant.
There are four significant digits in all.

d. 48 052.36 g/cm^3
The zero is significant.
There are seven significant digits in all.

e. 0.006048 m
The circled zeros are not significant.
There are four significant digits in all.

f. 4.306 00 L
The zeros are significant.
There are six significant digits in all.

g. 0.346 800 cm^2
The circled zeros is not significant.
There are six significant digits in all.

h. 0.000300 kg
The circled zeros are not significant.
There are three significant digits in all.

i. 6.70×10^4 mm
The zero is significant.
There are three significant digits in all.

j. 4.03×10^5 t (tonne)
The zero is significant.
There are three significant digits in all.

k. 6.009×10^{-2} m^3
The zeros are significant.
There are four significant digits in all.

l. 0.047 cm
The circled zeros are not significant.
There are two significant digits in all.

15. a. Add.

$$\begin{array}{r} 3.37\ \ \ g \\ 9.475\ g \\ 21.2\ \ \ \ g \\ \hline 34.045\ g \end{array}$$

34.045 g ≐ 34.0 g (three significant digits)

b. Add.

$$\begin{array}{r} 0.05\ \ L \\ 1.1\ \ \ L \\ 9.905\ L \\ \hline 11.055\ L \end{array}$$

11.055 L ≐ 11 L (two significant digits)

c. Subtract.

$$\begin{array}{r} 200.0\ \ m \\ 10.67\ m \\ \hline 189.33\ m \end{array}$$

189.33 m ≐ 189.3 m (four significant digits)

d. Subtract.

$$\begin{array}{r} 0.055\ \text{kg} \\ \underline{0.01\ \ \text{kg}} \\ 0.045\ \text{kg} \doteq 0.05\ \text{kg} \end{array}\ \text{(one significant digit)}$$

e. Multiply.

$$\begin{aligned}(3\ \text{g/mol})(6.91\ \text{mol}) &= 20.73\ \text{g} \\ &\doteq 20\ \text{g}\ \ \text{(one significant digit)}\end{aligned}$$

f. Multiply.

$$\begin{aligned}(0.035\ \text{g})(31°\text{C})(4.19\ \text{J/g°C}) &= 4.546\,15\ \text{J} \\ &\doteq 4.5\ \text{J}\ \ \text{(two significant digits)}\end{aligned}$$

g. Divide.

$$\begin{aligned}\frac{12.6\ \text{g}}{12.01\ \text{g/mol}} &= 1.049\,125\,792\ \text{mol} \\ &\doteq 1.05\ \text{mol}\ \ \text{(three significant digits)}\end{aligned}$$

h. Divide.

$$\begin{aligned}\frac{2.00\ \text{g}}{0.019\ \text{g/cm}^3} &= 105.263\,1579\ \text{cm}^3 \\ &\doteq 110\ \text{cm}^3\ \ \text{(two significant digits)}\end{aligned}$$

Exploring Topic 5

Activity 1

Add and subtract polynomials.

1. a. $(7a,\ 8a,\ 13a)$, $(6b^2,\ 12b^2,\ 13b^2)$, $(9ap,\ 3ap)$, $(ap^2,\ 3ap^2)$

   b. $(13,\ 2,\ 4)$, $(4x,\ 6x,\ 25x)$, $(7p,\ 5p)$, $(5pw^2)$, $(23pq)$

2. a. $$\begin{aligned}5x-3y+6x+5y &= 5x+6x-3y+5y \\ &= 11x+2y\end{aligned}$$

   b. $$\begin{aligned}4z+5c-7z^2+7c &= 5c+7c-7z^2+4z \\ &= 12c-7z^2+4z\end{aligned}$$

   c. $$\begin{aligned}6d^2+4d-3+5d^2-3d+1 &= 6d^2+5d^2+4d-3d-3+1 \\ &= 11d^2+d-2\end{aligned}$$

d. $(4e^4-5e^2+3)-(e^4-3e^2+6)$

$$=4e^4-5e^2+3-e^4+3e^2-6$$
$$=4e^4-e^4-5e^2+3e^2+3-6$$
$$=3e^4-2e^2-3$$

e. $f^3+17+(f^2-5f+2)+(3f^3-f^2-3)$

$$=f^3+17+f^2-5f+2+3f^3-f^2-3$$
$$=f^3+3f^3+17+2-3+f^2-f^2-5f$$
$$=4f^3-5f+16$$

3. a. Add.

$$\begin{array}{r} 4x^2-3x+4 \\ \underline{2x^2+5x-7} \\ 6x^2+2x-3 \end{array}$$

b. Add.

$$\begin{array}{l} 3x^2-3y+x-14 \\ \underline{2x^2+2y^2+3x+9} \end{array}$$

Rewrite as

$$\begin{array}{rrrrr} 3x^2 & & -3y & +x & -14 \\ 2x^2 & +2y^2 & & +3x & +9 \\ \hline 5x^2 & +2y^2 & -3y & +4x & -5 \end{array}$$

c. Subtract.

$$\begin{array}{r} 3n^2-4n+13 \\ \underline{5n^2+n-8} \\ -2n^2-5n+21 \end{array}$$

(Remember to change the signs when you are subtracting.)

d. Subtract.

$$\begin{array}{l} 3m^2+5w^2-3w+24 \\ \underline{3m^2-6m+4w-12} \end{array}$$

Rewrite as

$$\begin{array}{rrrrr} 3m^2 & +5w^2 & -3w & & +24 \\ 3m^2 & & +4w & -6m & -12 \\ \hline \end{array}$$

$5w^2-7w+6m+36$ or $5w^2+6m-7w+36$

4. a. $$\begin{aligned}(p+q+r)-(p-q-r)&=p+q+r-p+q+r\\&=p-p+q+q+r+r\\&=2q+2r\\&=2(4)+2(2)\\&=8+4\\&=12\end{aligned}$$

b. $$\begin{aligned}(2p+q-3r)+(3q-r-2p)&=2p+q-3r+3q-r-2p\\&=2p-2p+q+3q-3r-r\\&=4q-4r\\&=4(4)-4(2)\\&=16-8\\&=8\end{aligned}$$

c. $$\begin{aligned}&\left(p^2-2pq+q^2\right)-\left(3p^2+3pq-3q^2\right)\\&=p^2-2pq+q^2-3p^2-3pq+3q^2\\&=p^2-3p^2-2pq-3pq+q^2+3q^2\\&=-2p^2-5pq+4q^2\\&=-2(-1)^2-5(-1)(4)+4(4)^2\\&=-2(1)+20+4(16)\\&=-2+20+64\\&=82\end{aligned}$$

d. $$\begin{aligned}&\left(pqr+p^2qr^2\right)-\left(3pqr-3p^2qr^2\right)+\left(2pqr+4p^2qr^2\right)\\&=pqr+p^2qr^2-3pqr+3p^2qr^2+2pqr+4p^2qr^2\\&=pqr-3pqr+2pqr+p^2qr^2+3p^2qr^2+4p^2qr^2\\&=8p^2qr^2\\&=8(-1)^2(4)(2)\\&=8(1)(4)(4)\\&=128\end{aligned}$$

5. The two terms must be like. The literal coefficients must be identical and the exponents for these coefficients must be the same.

6. Like terms are lined up vertically. The numerical coefficients in each column or stack are added.

7. Let p be the number of people working on a project.
Let h be the number of hours spent on a project.
The amount spent in Department 1 is $\$(16p+9h)$.
The amount spent in Department 2 is $\$(9p+2h)$.
The amount spent in Department 3 is $\$(22p+27h)$.
What is the expenditure for each project?

$$\begin{array}{r}16p+\ \ 9h\\9p+\ \ 2h\\22p+27h\\\hline 47p+38h\end{array}$$

The total expense for the project will be $\$(47p+38h)$.

Extra Help

1. $8x^2+2x-5x+7=8x^2-3x+7$ This matches with L.
2. $4-3x^2-9x-7+x^2=-2x^2-9x-3$ This matches with A.
3. $-5x+8-4x^2-4x+2x^2=-2x^2-9x+8$ This matches with S.
4. $x^2-(-3x)+4+7x^2-8x-6=8x^2-5x-2$ This matches with E.
5. $-x-5x+\left(-3x^2\right)-9-2x+7=-3x^2-8x-2$ This matches with D.
6. $-7+x^3-5x^2+4x-5x+3=x^3-5x^2-x-4$ This matches with U.
7. $4x^3+6x^2+6x-1+5x^3-x^2-(-9)=9x^3+5x^2+6x+8$ This matches with M.
8. $-7x+5x^2-5x^3+8x+3x^2-7x^3+x^3=-11x^3+8x^2+x$ This matches with C.
9. $6x^3+(-2)-(-2x)-5x^3-4x^2+x+4x^2+15=x^3+3x+13$ This matches with H.
10. $6x^5-2x^4+6x^3-12x^5-6x^4+9x^3=-6x^5-8x^4+15x^3$ This matches with T.
11. $8ab-3b^2+2a^2-4ab+4b^2=2a^2+4ab+b^2$ This matches with B.
12. $5a^2b+9ab^2-2a^2b-13ab^2=3a^2b-4ab^2$ This matches with V.
13. $3a^3+b^3-6a^2b-a^3+6ab^2+a^2b=2a^3-5a^2b+6ab^2+b^3$ This matches with O.
14. $a^2b^2+a^2b-a^3-ab^2+a^2b-b^3-a^2b^2-b^3=-a^3+2a^2b-ab^2-2b^3$ This matches with R.

The solution to Why Did the Donkey Get a Passport is

3	13	9	4	8	13	6	1	5	11	4	8	13	7	4	2	10	14	2	12	4	1	11	6	14	14	13
S	O	H	E	C	O	U	L	D	B	E	C	O	M	E	A	T	R	A	V	E	L	B	U	R	R	O

So he could become a travel burro (bureau).

Extensions

1. $(7x+4)-(2x+9)=7x+4-2x-9$

   $=5x-5$ This matches with L.

2. $(3x+12)-(5x-6)=3x+12-5x+6$

   $=-2x+18$ This matches with H.

3. $\left(-4x^2+10\right)-\left(6x^2-9\right)=-4x^2+10-6x^2+9$

   $=-10x^2+19$ This matches with E.

4. $\left(2x^2+3x+8\right)-\left(x^2+5x-1\right)=2x^2+3x+8-x^2-5x+1$

   $=x^2-2x+9$ This matches with R.

5. $\left(-x^2+9x-2\right)-\left(9x^2-4x+4\right)=-x^2+9x-2-9x^2+4x-4$

   $=-10x+13x-6$ This matches with C.

6. $\left(3x^2+7x+1\right)-\left(8+5x+x^2\right)=3x^2+7x+1-8-5x-x^2$

   $=2x^2+2x-7$ This matches with Y.

7. $\left(4x^3+6x^2-8x\right)-\left(x^3-2x^2+12x\right)=4x^3+6x^2-8x-x^3+2x^2-12x$

   $=3x^3+8x^2-20x$ This matches with O.

(8) $\left(x^3+2x^2+5x\right)-\left(3x^2-x-7\right)=x^3+2x^2+5x-3x^2+x+7$

$=x^3-x^2+6x+7$ This matches with (I).

(9) $\left(x^4+8x^2-1\right)-\left(x^2-3x^3+x^4\right)=x^4+8x^2-1-x^2+3x^3-x^4$

$=3x^3+7x^2-1$ This matches with (P).

(10) $\left(5x^4-2x^2\right)-\left(3x-2x^2-4x^3+6x^4\right)=5x^4-2x^2-3x+2x^2+4x^3-6x^4$

$=-x^4+4x^3-3x$ This matches with (S).

(11) $\left(3x^2+7xy-2y^2\right)-\left(x^2-6xy+2y^2\right)=3x^2+7xy-2y^2-x^2+6xy-2y^2$

$=2x^2+13xy-4y^2$ This matches with (A).

(12) $\left(-x^2-9xy+5y^2\right)-\left(4x^2-2xy-y^2\right)=-x^2-9xy+5y^2-4x^2+2xy+y^2$

$=-5x^2-7xy+6y^2$ This matches with (T).

(13) $\left(4x^2y-3xy^2\right)-\left(3x^2y-8xy^2\right)=4x^2y-3xy^2-3x^2y+8xy^2$

$=x^2y+5xy^2$ This matches with (N).

The solution to Daffynition Decoder is

1. Romantic: **An Italian insect** (Roman-tic)

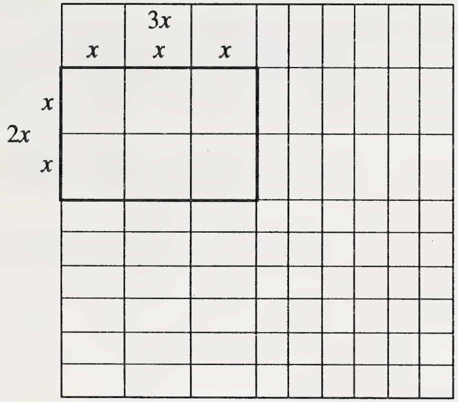
2. American: **A happy container** (A-meri-can)

Exploring Topic 6

Activity 1

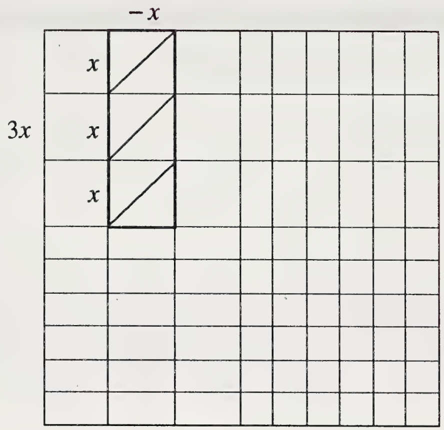
Multiply polynomials by monomials.

1. a.



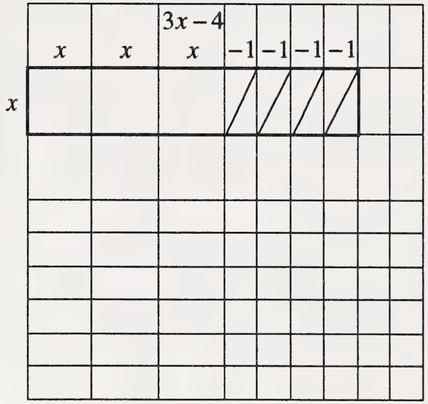
$(2x)(3x) = 6x^2$

b.



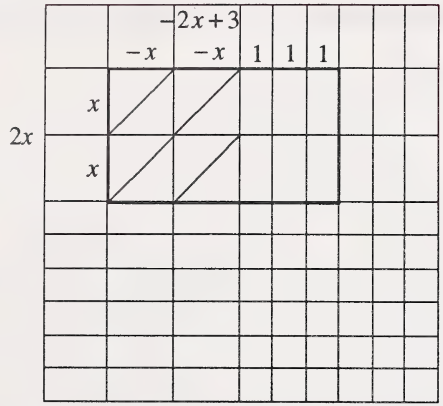
$(3x)(-x) = -3x^2$

c.



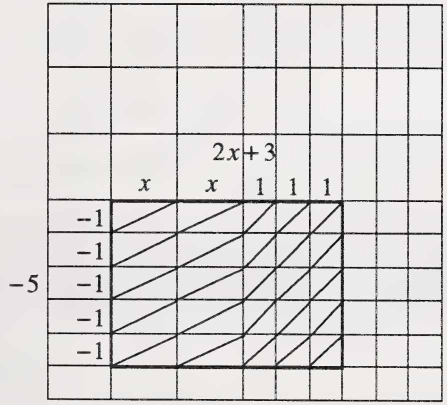
$(x)(3x-4) = 3x^2 - 4x$

d.



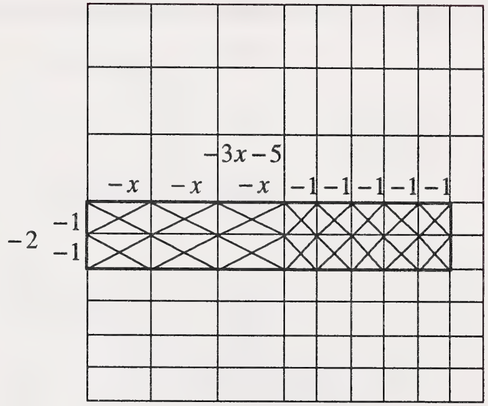
$(2x)(-2x+3)=-4x^2+6x$

e.



$-5(2x+3)=-10x-15$

f.



$-2(-3x-5)=6x+10$

2. a. $(3x^2)(5x)=(3\times5)(x^2\times x)$

$=15x^3$

b. $(15a^3)(2a)=(15\times2)(a^3\times a)$

$=30a^4$

c. $(5y^3)(8y^2)(2y)=(5\times8\times2)(y^3\times y^2\times y)$

$=80y^6$

d. $(5w^4)(12w^5)(w^2) = (5\times12\times1)(w^4\times w^5\times w^2)$
$= 60w^{11}$

e. $(6y^2)(3xy)(5x^2y^3) = (6\times3\times5)(x\times x^2)(y^2\times y\times y^3)$
$= 90x^3y^6$

f. $(18xy^2)(9x^2y)(3xy) = (18\times9\times3)(x\times x^2\times x)(y^2\times y\times y)$
$= 486x^4y^4$

g. $5s(s-4) = (5s)(s)-(5s)(4)$
$= 5s^2-20s$

h. $3t(9t-4) = (3t)(9t)-(3t)(4)$
$= 27t^2-12t$

i. $4t(t^2+2s) = (4t)(t^2)+(4t)(2s)$
$= 4t^3+8st$

j. $2m(3m^3-5n^2) = (2m)(3m^3)-(2m)(5n^2)$
$= 6m^4-10mn^2$

k. $4t(3t^2-16t+13) = (4t)(3t^2)-(4t)(16t)+(4t)(13)$
$= 12t^3-64t^2+52t$

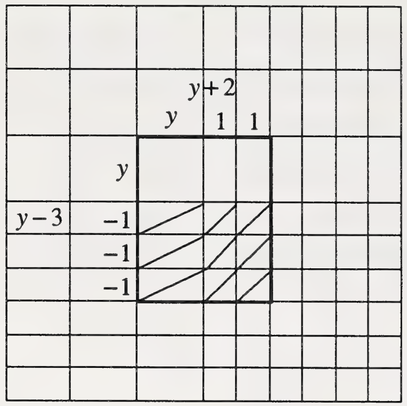
l. $12h(3h^2-5h+3) = (12h)(3h^2)-(12h)(5h)+(12h)(3)$
$= 36h^3-60h^2+36h$

3. Use the distributive property. Multiply each term of the polynomial by the monomial.

Activity 2

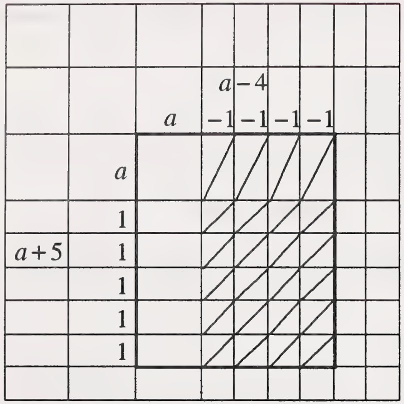
Multiply binomials by binomials.

1. a.



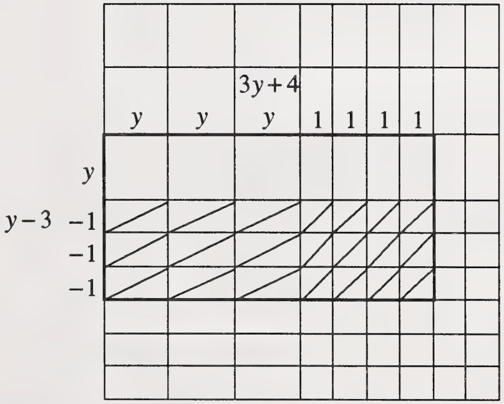
$(y-3)(y+2) = y^2-3y+2y-6$
$= y^2-y-6$

b.



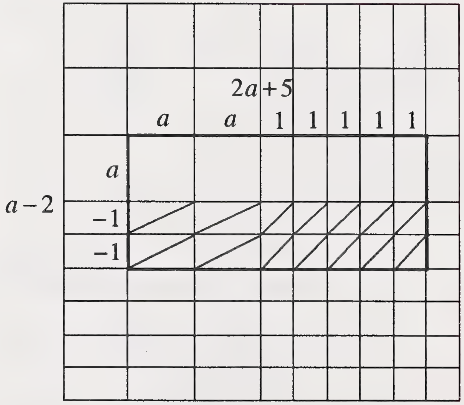
$$(a+5)(a-4) = a^2 + 5a - 4a - 20$$
$$= a^2 + a - 20$$

c.



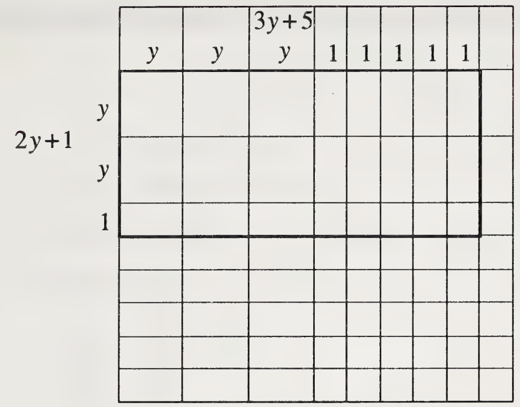
$$(y-3)(3y+4) = 3y^2 - 9y + 4y - 12$$
$$= 3y^2 - 5y - 12$$

d.



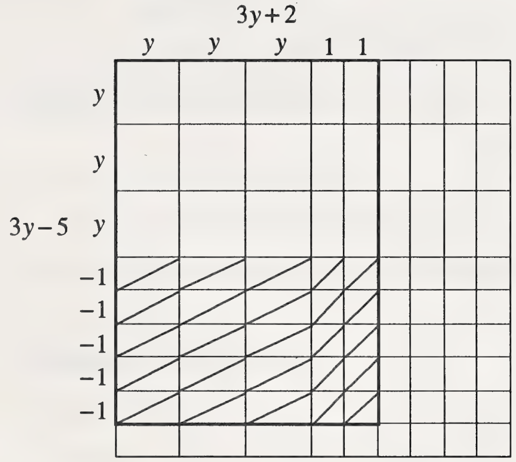
$$(a-2)(2a+5) = 2a^2 - 4a + 5a - 10$$
$$= 2a^2 + a - 10$$

e.



$$(2y+1)(3y+5) = 6y^2 + 3y + 10y + 5$$
$$= 6y^2 + 13y + 5$$

f.



$$(3y-5)(3y+2) = 9y^2 - 15y + 6y - 10$$
$$= 9y^2 - 9y - 10$$

2. a. $(w-3)(w+4) = (w)(w) + (w)(4) + (-3)(w) + (-3)(4)$
$$= w^2 + 4w - 3w - 12$$
$$= w^2 + w - 12$$

b. $(t+5)(t-7) = (t)(t) + (t)(-7) + (5)(t) + (5)(-7)$
$$= t^2 - 7t + 5t - 35$$
$$= t^2 - 2t - 35$$

c. $(2q+3)(q+6) = (2q)(q) + (2q)(6) + (3)(q) + (3)(6)$
$$= 2q^2 + 12q + 3q + 18$$
$$= 2q^2 + 15q + 18$$

d. $(2w-8)(w-4) = (2w)(w) + (2w)(-4) + (-8)(w) + (-8)(-4)$
$$= 2w^2 - 8w - 8w + 32$$
$$= 2w^2 - 16w + 32$$

e. $(3r+4)(5r-7) = (3r)(5r) + (3r)(-7) + (4)(5r) + (4)(-7)$
$$= 15r^2 - 21r + 20r - 28$$
$$= 15r^2 - r - 28$$

f. $(10m+7)(2m-5)$

$=(10m)(2m)+(10m)(-5)+(7)(2m)+(7)(-5)$

$=20m^2-50m+14m-35$

$=20m^2-36m-35$

g. $(3d+7t)(d-3t)=(3d)(d)+(3d)(-3t)+(7t)(d)+(7t)(-3t)$

$=3d^2-9dt+7dt-21t^2$

$=3d^2-2dt-21t^2$

h. $(7s+5t)(7s-5t)=(7s)(7s)+(7s)(-5t)+(5t)(7s)+(5t)(-5t)$

$=49s^2-35st+35st-25t^2$

$=49s^2-25t^2$

i. $(3m-2n)(3m+4)$

$=(3m)(3m)+(3m)(4)+(-2n)(3m)+(-2n)(4)$

$=9m^2+12m-6mn-8n$

j. $(5n-3m)(5n+4)$

$=(5n)(5n)+(5n)(4)+(-3m)(5n)+(-3m)(4)$

$=25n^2+20n-15mn-12m$

k. $\left(2m^2-m\right)\left(3m^2+m\right)$

$=\left(2m^2\right)\left(3m^2\right)+\left(2m^2\right)(m)+(-m)\left(3m^2\right)+(-m)(m)$

$=6m^4+2m^3-3m^3-m^2$

$=6m^4-m^3-m^2$

l. $\left(6w^3+w\right)\left(2w^2+3w\right)$

$=\left(6w^3\right)\left(2w^2\right)+\left(6w^3\right)(3w)+(w)\left(2w^2\right)+(w)(3w)$

$=12w^5+18w^4+2w^3+3w^2$

3. a. $(x-2)(x+2)=(x)(x)+(x)(2)+(-2)(x)+(-2)(2)$

$=x^2+2x-2x-4$

$=x^2-4$

This is a Difference of Squares.

b. $(b+9)(b-9)=(b)(b)+(b)(-9)+(9)(b)+(9)(-9)$

$=b^2-9b+9b-81$

$=b^2-81$

This is a Difference of Squares.

c. $(3b-4)(3b+4)=(3b)(3b)+(3b)(4)+(-4)(3b)+(-4)(4)$

$$=9b^2+12b-12b-16$$
$$=9b^2-16$$

This is a Difference of Squares.

d. $(4n^2-8)(4n^2+8)$

$$=(4n^2)(4n^2)+(4n^2)(8)+(-8)(4n^2)+(-8)(8)$$
$$=16n^2+32n^2-32n^2-64$$
$$=16n^4-64$$

This is a Difference of Squares.

e. $(x+2)(x+2)=(x)(x)+(x)(2)+(2)(x)+(2)(2)$

$$=x^2+2x+2x+4$$
$$=x^2+4x+4$$

This is a Perfect Square Trinomial.

f. $(2x-7)(2x-7)$

$$=(2x)(2x)+(2x)(-7)+(-7)(2x)+(-7)(-7)$$
$$=4x^2-14x-14x+49$$
$$=4x^2-28x+49$$

This is a Perfect Square Trinomial.

g. $(3q-2p)(3q-2p)$

$$=(3q)(3q)+(3q)(-2p)+(-2p)(3q)+(-2p)(-2p)$$
$$=9q^2-6pq-6pq+4p^2$$
$$=9q^2-12pq+4p^2$$
$$=4p^2-12pq+9q^2$$

This is a Perfect Square Trinomial.

h. $(3m^4+2n^2)(3m^4+2n^2)$

$$=(3m^4)(3m^4)+(3m^4)(2n^2)+(2n^2)(3m^4)+(2n^2)(2n^2)$$
$$=9m^8+6m^4n^2+6m^4n^2+4n^4$$
$$=9m^8+12m^4n^2+4n^4$$

This is a Perfect Square Trinomial.

4. a. $(3c+8)^2=(3c+8)(3c+8)$

$$=(3c)(3c)+(3c)(8)+(8)(3c)+(8)(8)$$
$$=9c^2+24c+24c+64$$
$$=9c^2+48c+64$$

b. $$\begin{aligned}(2b-3)^2-(3b+4)(2b+3) &= (2b-3)(2b-3)-[(3b)(2b)+(3b)(3)+(4)(2b)+(4)(3)] \\ &= (2b)(2b)+(2b)(-3)+(-3)(2b)+(-3)(-3)-\left[6b^2+9b+8b+12\right] \\ &= 4b^2-6b-6b+9-\left[6b^2+17b+12\right] \\ &= 4b^2-12b+9-\left[6b^2+17b+12\right] \\ &= 4b^2-12b+9-6b^2-17b-12 \\ &= 4b^2-6b^2-12b-17b+9-12 \\ &= -2b^2-29b-3\end{aligned}$$

c. $$\begin{aligned}(2x+y)(2x+y)-(x-2y)^2 &= (2x)(2x)+(2x)(y)+(y)(2x)+(y)(y)-[(x-2y)(x-2y)] \\ &= 4x^2+2xy+2xy+y^2-[(x)(x)+(x)(-2y)+(-2y)(x)+(-2y)(-2y)] \\ &= 4x^2+4xy+y^2-\left[x^2-2xy-2xy+4y^2\right] \\ &= 4x^2+4xy+y^2-\left[x^2-4xy+4y^2\right] \\ &= 4x^2+4xy+y^2-x^2+4xy-4y^2 \\ &= 4x^2-x^2+4xy+4xy+y^2-4y^2 \\ &= 3x^2+8xy-3y^2\end{aligned}$$

d. $$\begin{aligned}3d(2d-3)^2+(-5d^2+4) &= 3d(2d-3)(2d-3)+(-5d^2+4)\\ &= 3d[(2d)(2d)+(2d)(-3)+(-3)(2d)+(-3)(-3)]+(-5d^2+4)\\ &= 3d[4d^2-6d-6d+9]+(-5d^2+4)\\ &= 3d(4d^2-12d+9)+(-5d^2+4)\\ &= (3d)(4d^2)+(3d)(-12d)+(3d)(9)+(-5d^2+4)\\ &= 12d^3-36d^2+27d+(-5d^2+4)\\ &= 12d^3-36d^2+27d-5d^2+4\\ &= 12d^3-36d^2-5d^2+27d+4\\ &= 12d^3-41d^2+27d+4\end{aligned}$$

5. Since both factors are identical, use only one of the factors. Square the first term in the factor, square the second term of the factor, and multiply the product of these two factors by two. Express each product as a trinomial. If the sign between the original two terms is positive, the second term is positive; if the sign is negative, the second term is negative.

Extra Help

1. $$\begin{aligned}5(2n^2+n) &= 5(2n^2)+5(n)\\ &= 10n^2+5n\end{aligned}$$ This matches with S.

2. $$\begin{aligned}3n(8n^2-2n) &= 3n(8n^2)+3n(-2n)\\ &= 24n^3-6n^2\end{aligned}$$ This matches with L.

(3) $n^2(4n-3)=n^2(4n)+n^2(-3)$

$=4n^3-3n^2$ This matches with (U).

(4) $-2n(4+5n^3)=-2n(4)-2n(5n^3)$

$=-8n-10n^4$ This matches with (A).

(5) $-6n^2(4n^2-9)=-6n^2(4n^2)-6n^2(-9)$

$=-24n^4+54n^2$ This matches with (R).

(6) $4a(a^2-2a+3)=4a(a^2)+4a(-2a)+4a(3)$

$=4a^3-8a^2+12a$ This matches with (G).

(7) $-2a^2(9-a-4a^2)=-2a^2(9)-2a^2(-a)-2a^2(-4a^2)$

$=-18a^2+2a^3+8a^4$ This matches with (H).

(8) $a^2b(a^2-b^2)=a^2b(a^2)+a^2b(-b^2)$

$=a^4b-a^2b^3$ This matches with (A).

(9) $-3ab^2(a^3b^2-2a^2b)=-3ab^2(a^3b^2)-3ab^2(-2a^2b)$

$=-3a^4b^4+6a^3b^3$ This matches with (L).

(10) $2ab(a^2+4ab-3b^2)=2ab(a^2)+2ab(4ab)+2ab(-3b^2)$

$=2a^3b+8a^2b^2-6ab^3$ This matches with (E).

(11) $x^2y(2x^2-4xy+y^2)=x^2y(2x^2)+x^2y(-4xy)+x^2y(y^2)$

$=2x^4y-4x^3y^2+x^2y^3$ This matches with (Y).

(12) $-2xy^2(2x^4-5x^2y^2-3y^4)=-2xy^2(2x^4)-2xy^2(-5x^2y^2)-2xy^2(-3y^4)$

$=-4x^5y^2+10x^3y^4+6xy^6$ This matches with (N).

(13) $4x^3y(-x^2y+2xy-5xy^2)=4x^3y(-x^2y)+4x^3y(2xy)+4x^3y(-5xy^2)$

$=-4x^5y^2+8x^4y^2-20x^4y^3$ This matches with (E).

(14) $-x^2y^3(7xy^3-x^2y^2+3x^3y)=-x^2y^3(7xy^3)-x^2y^3(-x^2y^2)-x^2y^3(3x^3y)$

$=-7x^3y^6+x^4y^5-3x^5y^4$ This matches with (I).

(15) $3x^2y^2(2x^4y^2+3x^2y-1)=3x^2y^2(2x^4y^2)+3x^2y^2(-3x^2y)+3x^2y^2(-1)$

$=6x^6y^4+9x^4y^3-3x^2y^2$ This matches with (F).

The solution to What Did The Girl Mushroom Say About The Boy Mushroom After Their First Date? is

He's really a fungi (fun guy).

Extensions

1. $(x+3)(x+5) = x^2 + 5x + 3x + 15$
$= x^2 + 8x + 15$

This matches with E.

2. $(x+2)(x+9) = x^2 + 9x + 2x + 18$
$= x^2 + 11x + 18$

This matches with U.

3. $(x-8)(x+1) = x^2 + x - 8x - 8$
$= x^2 - 7x - 8$

This matches with B.

4. $(x-3)(x-6) = x^2 - 6x - 3x + 18$
$= x^2 - 9x + 18$

This matches with A.

5. $(2x+9)(x-2) = 2x^2 - 4x + 9x - 18$
$= 2x^2 + 5x - 18$

This matches with N.

6. $(3x+1)(2x+4) = 6x^2 + 12x + 2x + 4$
$= 6x^2 + 14x + 4$

This matches with S.

7. $(4a-7)(3a-2) = 12a^2 - 8a - 21a + 14$
$= 12a^2 - 29a + 14$

This matches with O.

8. $(2a+5)(2a-5) = 4a^2 - 10a + 10a - 25$
$= 4a^2 - 25$

This matches with R.

9. $(6a-1)(2a+4) = 12a^2 + 24a - 2a - 4$
$= 12a^2 + 22a - 4$

This matches with E.

10. $(a+2b)(4a+b) = 4a^2 + ab + 8ab + 2b^2$
$= 4a^2 + 9ab + 2b^2$

This matches with T.

11. $(5a+3b)(a-4b)=5a^2-20ab+3ab-12b^2$

$=5a^2-17ab-12b^2$

This matches with N.

12. $(3a-8b)(2a-b)=6a^2-3ab-16ab+8b^2$

$=6a^2-19ab+8b^2$

This matches with I.

13. $(n+2)(n^2+5n-3)=n^3+5n^2-3n+2n^2+10n-6$

$=n^3+7n^2+7n-6$

This matches with N.

14. $(3n-1)(2n^2+4n+4)=6n^3+12n^2+12n-2n^2-4n-4$

$=6n^3+10n^2+8n-4$

This matches with I.

15. $(2n+3)(6n^2-2n+1)=12n^3-4n^2+2n+18n^2-6n+3$

$=12n^3+14n^2-4n+3$

This matches with D.

16. $(4n-5)(n^2-7n-2)=4n^3-28n^2-8n-5n^2+35n+10$

$=4n^3-33n^2+27n+10$

This matches with R.

17. $(3n-4)(4n^2+2n+3)=12n^3+6n^2+9n-16n^2-8n-12$

$=12n^3-10n^2+n-12$

This matches with A.

18. $(n+8)(6n^2-n-4)=6n^3-n^2-4n+48n^2-8n-32$

$=6n^3+47n^2-12n-32$

This matches with T.

~~B~~	$x^2 - 7x - 8$
~~E~~	$x^2 + 8x + 15$
~~S~~	$6x^2 + 14x + 4$
I	$6x^2 + 7x + 4$
~~A~~	$x^2 - 9x + 18$
~~U~~	$x^2 + 11x + 18$
T	$x^2 - 13x + 18$
~~N~~	$2x^2 + 5x - 18$
~~T~~	$4a^2 + 9ab + 2b^2$
~~I~~	$6a^2 - 19ab + 8b^2$
S	$5a^2 - 11ab - 12b^2$
~~E~~	$12a^2 + 22a - 4$
~~R~~	$4a^2 - 25$

A	$4a^2 + 4ab + 3b^2$
~~N~~	$5a^2 - 17ab - 12b^2$
~~O~~	$12a^2 - 29a + 14$
~~T~~	$6n^3 + 47n^2 - 12n - 32$
C	$6n^3 + 44n^2 - 9n - 32$
~~R~~	$4n^3 - 33n^2 + 27n + 10$
~~I~~	$6n^3 + 10n^2 + 8n - 4$
H	$n^3 + 6n^2 + 9n - 6$
E	$12n^3 - 9n^2 - 2n - 12$
~~A~~	$12n^3 - 10n^2 + n - 12$
~~N~~	$n^3 + 7n^2 + 7n - 6$
W	$4n^3 - 30n^2 + 21n + 10$
~~D~~	$12n^3 + 14n^2 - 4n + 3$

The solution to Why Is a Stick of Gum Like a Sneeze? is

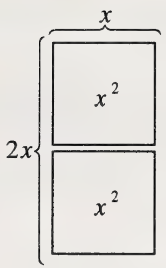
It's a chew (achoo).

Exploring Topic 7

Activity 1

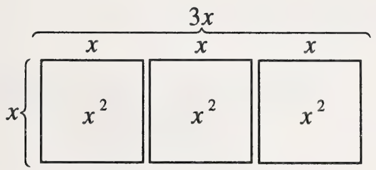
Divide polynomials by monomials.

1. a.



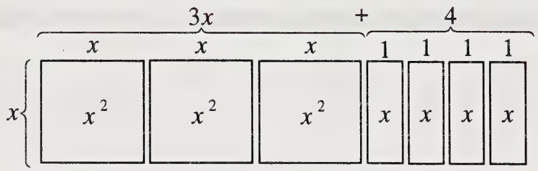
$2x^2 \div 2x = x$

b.



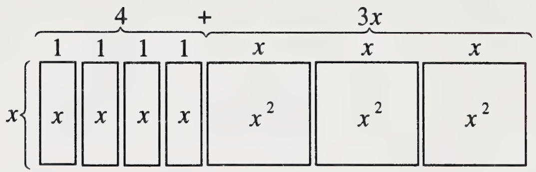
$3x^2 \div x = 3x$

c.



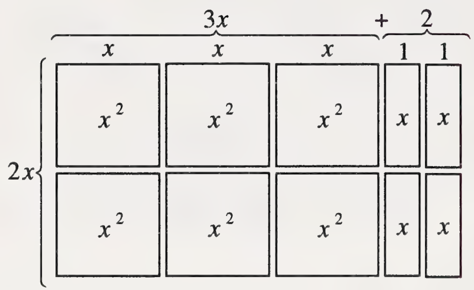
$(4x + 3x^2) \div x = 3x + 4$

or



$(4x + 3x^2) \div x = 4 + 3x$

d.



$(6x^2 + 4x) \div 2x = 3x + 2$

2\. a. $16m^3n^2 \div 12m^2n = \dfrac{16m^3n^2}{12m^2n}$

$= \dfrac{4}{3}mn$

b. $45st^3 \div \left(-30s^2t^2\right) = \dfrac{45st^3}{-30s^2t^2}$

$= \dfrac{-3t}{2s}$ or $\dfrac{-3}{2}s^{-1}t$

c. $\dfrac{36a^5b^2c^3}{18a^3bc^2} = \dfrac{36}{18} \times \dfrac{a^5}{a^3} \times \dfrac{b^2}{b} \times \dfrac{c^3}{c^2}$

$= 2a^2bc$

d. $\dfrac{-52mn^5}{-39m^2n^2} = \dfrac{-52}{-39} \times \dfrac{m}{m^2} \times \dfrac{n^5}{n^2}$

$= \dfrac{4n^3}{3m}$

e. $\left(2n^3 + 16n^2 - 10n\right) \div 2n = \dfrac{2n^3}{2n} + \dfrac{16n^2}{2n} + \dfrac{-10n}{2n}$

$= n^2 + 8n - 5$

f. $\left(6p^3 + 15p^2q^2 - 9pq^3\right) \div 3q = \dfrac{6p^3}{3q} + \dfrac{15p^2q^2}{3q} - \dfrac{9pq^3}{3q}$

$= \dfrac{2p^3}{q} + 5p^2q - 3pq^2$

g. $\dfrac{3y^2 + 12y - 6}{3} = \dfrac{3y^2}{3} + \dfrac{12y}{3} - \dfrac{6}{3}$

$= y^2 + 4y - 2$

h. $\dfrac{16a^4 - 8a^2b^2}{8a} = \dfrac{16a^4}{8a} - \dfrac{8a^2b^2}{8a}$

$= 2a^3 - ab^2$

3\. a.

$$\begin{array}{r@{}l} & 4x+3 \\ 4x\!\!\!& \overline{)16x^2+12x+15} \\ & \underline{16x^2} \\ & 0+12x \\ & \underline{12x} \\ & 0+15 \\ & \underline{0} \\ & 15 \end{array}$$

$\left(16x^2 + 12x + 15\right) \div 4x = 4x + 3$ r: 5

b. $$\begin{array}{r} 3x+1 \\ 5x\overline{)15x^2+5x+16} \\ \underline{15x^2} \\ 0+5x \\ \underline{5x} \\ 0+16 \\ \underline{0} \\ 16 \end{array}$$

$\left(15x^2+5x+16\right)\div 5x = 3x+1 \quad$ r: 16

4. The area of the pool is $\left(9x^4-12x^3+6x^2\right)$ m^2.

The width of the pool is $3x^2$ m.
Since area is the length times the width, what is the length of the pool?

$A = l \times w \;\therefore \frac{A}{w} = l$

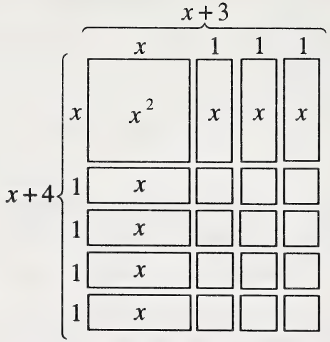
$$\begin{aligned} l &= \frac{A}{w} \\ &= \frac{9x^4-12x^3+6x^2}{3x^2} \\ &= \frac{9x^4}{3x^2} - \frac{12x^3}{3x^2} + \frac{6x^2}{3x^2} \\ &= 3x^2-4x+2 \end{aligned}$$

The length of the pool is $\left(3x^2-4x+2\right)$ m.

Activity 2

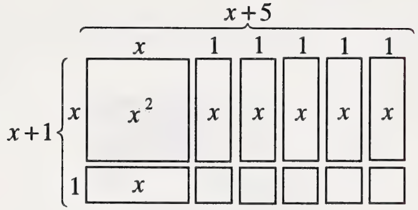
Divide polynomials by binomials of the form $ax+b$.

1. a.



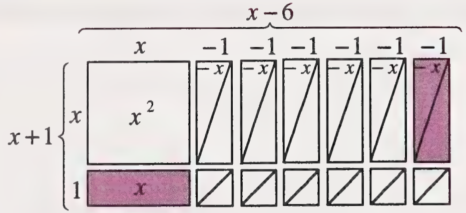
$\left(x^2+7x+12\right)\div(x+4) = x+3$

b.



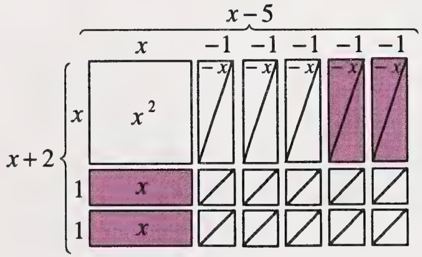
$\left(x^2+6x+5\right)\div(x+1) = x+5$

c.



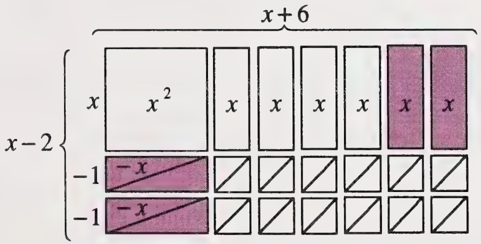
$(x^2 - 5x - 6) \div (x+1) = x - 6$

d.



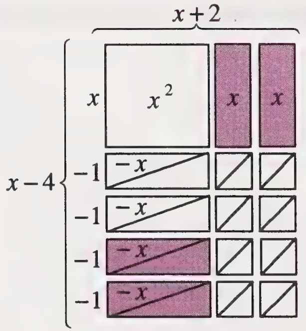
$(x^2 - 3x - 10) \div (x+2) = x - 5$

e.



$(x^2 + 4x - 12) \div (x-2) = x + 6$

f.



$(x^2 - 2x - 8) \div (x-4) = x + 2$

Note: In question c to f you cannot make a completed rectangle with the tiles that represent the polynomial.

Therefore, lay out the x^2-tile and the -1-tiles. Then put in the x-tiles and fill in the rectangle with negative and positive x-tiles as required. Remember $(-x)+x=0$; so you are not changing the polynomial when you add an equal number of postive and negative x-tiles.

2. a. $(x^2 + 10x + 24) \div (x+4) = \dfrac{x^2 + 10x + 24}{x+4}$

$$= \frac{x^2 + 4x + 6x + 24}{x+4}$$

$4 \times 6 = 24$

$4 + 6 = 10$

$$= \frac{x^2 + 4x}{x+4} + \frac{6x + 24}{x+4}$$

$$= \frac{x\cancel{(x+4)}}{\cancel{x+4}} + \frac{6\cancel{(x+4)}}{\cancel{x+4}}$$

$$= x + 6$$

b. $(x^2+10x-24)\div(x-2)=\dfrac{x^2+10x-24}{x-2}$

$$=\frac{x^2-2x+12x-24}{x-2}$$

$-2\times12=-24$

$-2+12=10$

$$=\frac{x^2-2x}{x-2}+\frac{12x-24}{x-2}$$

$$=\frac{x\cancel{(x-2)}}{\cancel{x-2}}+\frac{12\cancel{(x-2)}}{\cancel{x-2}}$$

$$=x+12$$

c. $(m^2-14m+33)\div(m-3)=\dfrac{m^2-14m+33}{m-3}$

$$=\frac{m^2-3m-11m+33}{m-3}$$

$-3\times11=-33$

$-3+-11=-14$

$$=\frac{m^2-3m}{m-3}-\frac{11m-33}{m-3}$$

$$=\frac{m\cancel{(m-3)}}{\cancel{m-3}}-\frac{11\cancel{(m-3)}}{\cancel{m-3}}$$

$$=m-11$$

d. $(n^2-9n-52)\div(n+4)=\dfrac{n^2-9n-52}{n+4}$

$$=\frac{n^2+4n-13n-52}{n+4}$$

$4\times-13=-52$

$4+-13=-9$

$$=\frac{n^2+4n}{n+4}-\frac{13n+52}{n+4}$$

$$=\frac{n\cancel{(n+4)}}{\cancel{n+4}}-\frac{13\cancel{(n+4)}}{\cancel{n+4}}$$

$$=n-13$$

e. $(a^2+10a-39)\div(a-3)=\dfrac{a^2+10a-39}{a-3}$

$$=\frac{a^2-3a+13a-39}{a-3}$$

$-3\times13=-39$

$-3+13=10$

$$=\frac{a^2-3a}{a-3}+\frac{13a-39}{a-3}$$

$$=\frac{a\cancel{(a-3)}}{\cancel{a-3}}+\frac{13\cancel{(a-3)}}{\cancel{a-3}}$$

$$=a+13$$

f. $(y^2+5y-66)\div(y-6)=\dfrac{y^2+5y-66}{y-6}$

$$=\frac{y^2-6y+11y-66}{y-6}$$

$-6\times 11=-66$

$-6+11=5$

$$=\frac{y^2-6y}{y-6}+\frac{11y-66}{y-6}$$

$$=\frac{y\cancel{(y-6)}}{\cancel{y-6}}+\frac{11\cancel{(y-6)}}{\cancel{y-6}}$$

$$=y+11$$

3. a.

$$\begin{array}{r}
2a-3 \\
a+10\overline{)2a^2+17a-15} \\
\underline{2a^2+20a} \\
-3a-15 \\
\underline{-3a-30} \\
15
\end{array}$$

$(2a^2+17a-15)\div(a+10)=2a-3 \quad$ r: 15

b.

$$\begin{array}{r}
3a-4 \\
a+44\overline{)3a^2+128a-315} \\
\underline{3a^2+132a} \\
-4a-315 \\
\underline{-4a-176} \\
-139
\end{array}$$

$(3a^2+128a-315)\div(a+44)=3a-4 \quad$ r: -139

4. The algebraic method depends on the division working out evenly. If the division does not work out evenly, then the algebraic method will not give an answer. Long division will always give an answer. Therefore, it would be beneficial to use long division before using the algebraic method.

Extra Help

1. $\dfrac{x^2+8x+15}{x+5}=\dfrac{x^2+5x+3x+15}{x+5}$

$$=\frac{x^2+5x}{x+5}+\frac{3x+15}{x+5}$$

$5\times 3=15$

$5+3=8$

$$=\frac{x\cancel{(x+5)}^{1}}{\cancel{(x+5)}_{1}}+\frac{3\cancel{(x+5)}^{1}}{\cancel{(x+5)}_{1}}$$

$$=x+3$$

② $$\frac{2x^2+3x-14}{x-2}=\frac{2x^2-4x+7x-14}{x-2}$$

$$=\frac{2x^2-4x}{x-2}+\frac{7x-14}{x-2}$$

$-4\times 7=-28$

$-4+7=3$

$$=\frac{2x\cancel{(x-2)}^{1}}{\cancel{(x-2)}_{1}}+\frac{7\cancel{(x-2)}^{1}}{\cancel{(x-2)}_{1}}$$

$$=2x+7$$

③ $$\frac{x^2-5x+8}{x-3}=\begin{array}{r} x-2 \\ x-3\overline{)x^2-5x+8} \\ \underline{x^2-3x} \\ -2x+8 \\ \underline{-2x+6} \\ 2 \end{array}$$

The quotient is $x-2+\frac{2}{x-3}$.

④ $$\frac{x^2-x+12}{x-6}=\begin{array}{r} x+5 \\ x-6\overline{)x^2-x+12} \\ \underline{x^2-6x} \\ 5x+12 \\ \underline{5x-30} \\ 42 \end{array}$$

The quotient is $x+5+\frac{42}{x-6}$.

⑤ $$\frac{3x^2-5x-11}{x+1}=\begin{array}{r} 3x-8 \\ x+1\overline{)3x^2-5x-11} \\ \underline{3x^2+3x} \\ -8x-11 \\ \underline{-8x-8} \\ -3 \end{array}$$

The quotient is $3x-8-\frac{3}{x+1}$.

⑥ $$\frac{x^2+1+8x}{x+4}=\begin{array}{r} x+4 \\ x+4\overline{)x^2+8x+1} \\ \underline{x^2+4x} \\ 4x+1 \\ \underline{4x+16} \\ -15 \end{array}$$

The quotient is $x+4-\frac{15}{x+4}$.

⑦ $$\frac{x^2+4}{x-3}=\begin{array}{r} x+3 \\ x-3\overline{)x^2+0x+4} \\ \underline{x^2-3x} \\ 3x+4 \\ \underline{3x-9} \\ 13 \end{array}$$

The quotient is $x+3+\frac{13}{x-3}$.

(8) $\dfrac{2x^2-3x-1}{2x+1} = 2x+1\overline{)2x^2-3x-1}$

$$
\begin{array}{r}
x-2 \\
2x+1\overline{)2x^2-3x-1} \\
\underline{2x^2+x} \\
-4x-1 \\
\underline{-4x-2} \\
1
\end{array}
$$

The quotient is $x-2+\frac{1}{2x+1}$.

(9) $\dfrac{6x^2-7x+5}{3x-5} = 3x-5\overline{)6x^2-7x+5}$

$$
\begin{array}{r}
2x+1 \\
3x-5\overline{)6x^2-7x+5} \\
\underline{6x^2-10x} \\
3x+5 \\
\underline{3x-5} \\
10
\end{array}
$$

The quotient is $2x+1+\frac{10}{3x-5}$.

TH $\cancel{x-2+\frac{2}{x-3}}$	HE $\cancel{x+3+\frac{13}{x-3}}$	AT $2x+2+\frac{6}{3x-5}$	ST $\cancel{x+5+\frac{42}{x-6}}$	SH $\cancel{2x+1+\frac{10}{3x-5}}$
RA $x-3+\frac{3}{2x+1}$	SK $\cancel{x+3}$	OT $\cancel{x-2+\frac{1}{2x+1}}$	IN $3x-6-\frac{5}{x+1}$	HI $x+4+\frac{9}{x-3}$
BE $\cancel{3x-8-\frac{3}{x+1}}$	TH $x+2-\frac{11}{x+4}$	HU $\cancel{2x+7}$	NT $\cancel{x+4-\frac{15}{x+4}}$	IM $x+7+\frac{33}{x-6}$

A	T	R	A	I	N	H	I	T	H	I	M

The solution to How Did the Hunter Get Hurt While Bending Over to Study Some Tracks? is **A train hit him.**

Extensions

1. $\dfrac{4x^2-4x+3}{2x-5} = 2x-5\overline{\big)4x^2-4x+3}$

$$\begin{array}{r} 2x+3 \\ 2x-5\overline{\big)4x^2-4x+3} \\ \underline{4x^2-10x} \\ 6x+3 \\ \underline{6x-15} \\ 18 \end{array}$$

The quotient is $2x+3+\frac{18}{2x-5}$.

2. $\dfrac{2x^2-20}{x+3} = x+3\overline{\big)2x^2+0x-20}$

$$\begin{array}{r} 2x-6 \\ x+3\overline{\big)2x^2+0x-20} \\ \underline{2x^2+6x} \\ -6x-20 \\ \underline{-6x-18} \\ -2 \end{array}$$

The quotient is $2x-6-\frac{2}{x+3}$.

3. $\dfrac{x^3+5x^2+4x-4}{x+2} = x+2\overline{\big)x^3+5x^2+4x-4}$

$$\begin{array}{r} x^2+3x-2 \\ x+2\overline{\big)x^3+5x^2+4x-4} \\ \underline{x^3+2x^2} \\ 3x^2+4x \\ \underline{3x^2+6x} \\ -2x-4 \\ \underline{-2x-4} \\ 0 \end{array}$$

The quotient is x^2+3x-2.

4. $\dfrac{1-7x^2+6x^3+17x}{3x-2} = 3x-2\overline{\big)6x^3-7x^2+17x+1}$

$$\begin{array}{r} 2x^2-x+5 \\ 3x-2\overline{\big)6x^3-7x^2+17x+1} \\ \underline{6x^3-4x^2} \\ -3x^2+17x \\ \underline{-3x^2+2x} \\ 15x+1 \\ \underline{15x-10} \\ 11 \end{array}$$

The quotient is $2x^2-x+5+\frac{11}{3x-2}$.

(5) $\dfrac{x^3-8}{x-2} = x-2\overline{\big)x^3+0x^2+0x-8}$

$$\begin{array}{r}
x^2+2x+4 \\
x-2\overline{\big)x^3+0x^2+0x-8} \\
\underline{x^3-2x^2} \\
2x^2+0x \\
\underline{2x^2-4x} \\
4x-8 \\
\underline{4x-8} \\
0
\end{array}$$

The quotient is x^2+2x+4.

(6) $\dfrac{x^3+9x^2-80}{x+4} = x+4\overline{\big)x^3+9x^2+0x-80}$

$$\begin{array}{r}
x^2+5x-20 \\
x+4\overline{\big)x^3+9x^2+0x-80} \\
\underline{x^3+4x^2} \\
5x^2+0x \\
\underline{5x^2+20x} \\
-20x-80 \\
\underline{-20x-80} \\
0
\end{array}$$

The quotient is $x^2+5x-20$.

(7) $\dfrac{6a^2+5ab-5b^2}{2a-b} = 2a-b\overline{\big)6a^2+5ab-5b^2}$

$$\begin{array}{r}
3a+4b \\
2a-b\overline{\big)6a^2+5ab-5b^2} \\
\underline{6a^2-3ab} \\
8ab-5b^2 \\
\underline{8ab-4b^2} \\
-b^2
\end{array}$$

The quotient is $3a+4b-\frac{b^2}{2a-b}$.

(8) $\dfrac{a^3+4a^2b+ab^2-2b^3}{a+b} = a+b\overline{\big)a^3+4a^2b+ab^2-2b^3}$

$$\begin{array}{r}
a^2+3ab-2b^2 \\
a+b\overline{\big)a^3+4a^2b+ab^2-2b^3} \\
\underline{a^3+a^2b} \\
3a^2b+ab^2 \\
\underline{3a^2b+3ab^2} \\
-2ab^2-2b^3 \\
\underline{-2ab^2-2b^3} \\
0
\end{array}$$

The quotient is $a^2+3ab-2b^2$.

D x^2+2x-7

A $3a+2b-\frac{8b^2}{2a-b}$

O $x^2+5x-18$

T $a^2+3ab-2b^2$

R $2x-6-\frac{2}{x+3}$

C $2x+3+\frac{18}{2x-5}$

S x^2+2x+4

H $x^2+5x-20$

U $2x^2-x-5+\frac{4}{3x-2}$

N $2x-6+\frac{7}{x+3}$

I x^2+3x-2

W $3a+4b-\frac{b^2}{2a-b}$

E $2x^2-x+5+\frac{11}{3x-2}$

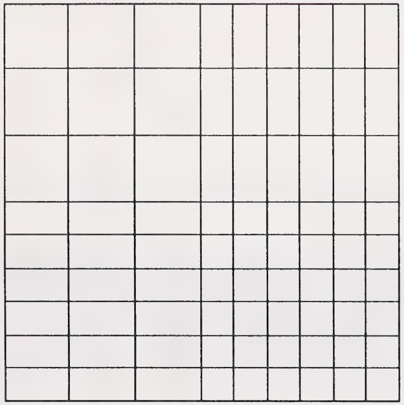
M $a^2+3ab-b^2+\frac{5b}{a+b}$

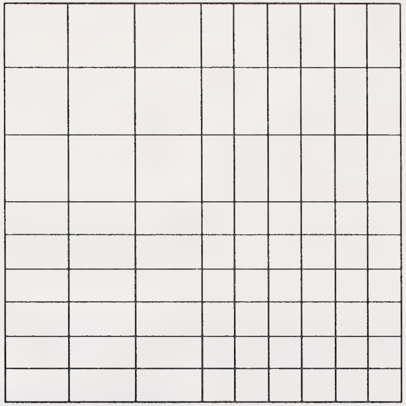
5	7	3	8	1	6	6	3	8	8	4	2	5
S	**W**	**I**	**T**	**C**	**H**	**H**	**I**	**T**	**T**	**E**	**R**	**S**

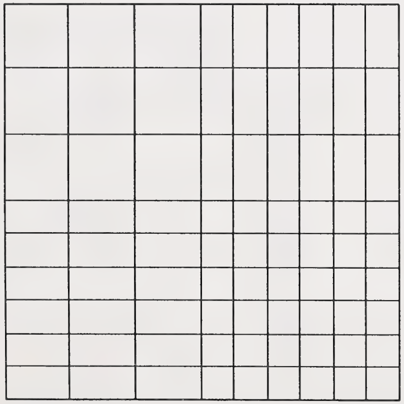
The solution to What Do They Call People Who Like To Turn The Lights On and Off is **Switch hitters**.

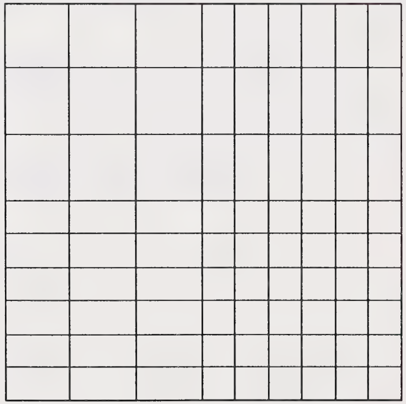
Appendix B
Grids and Tiles

Binomial Grids

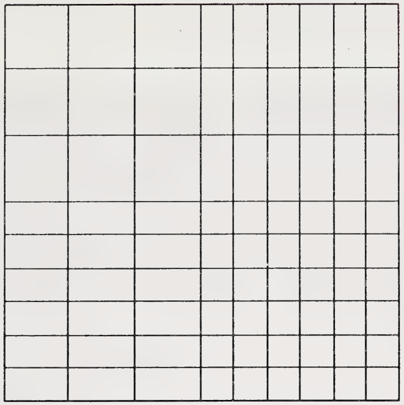


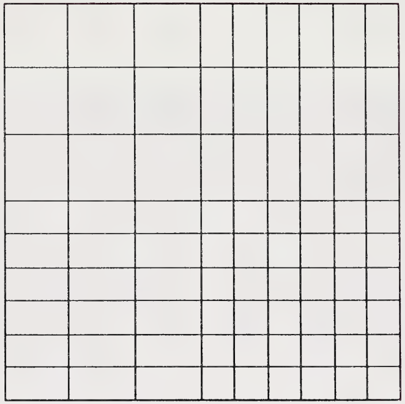


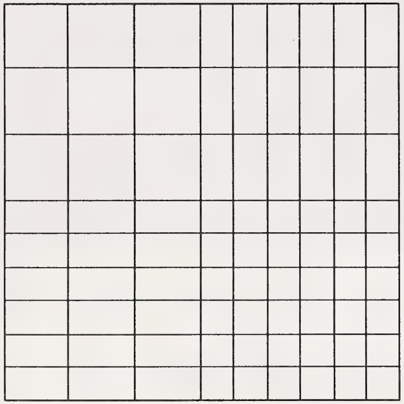


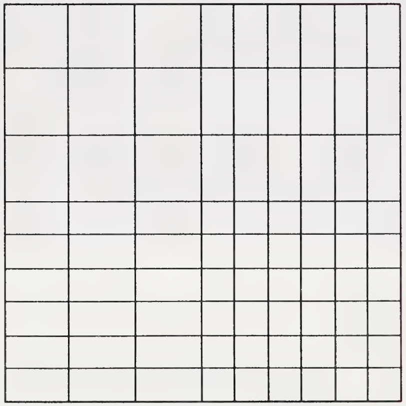


Binomial Grids

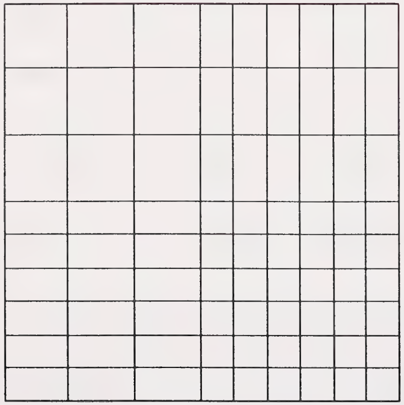


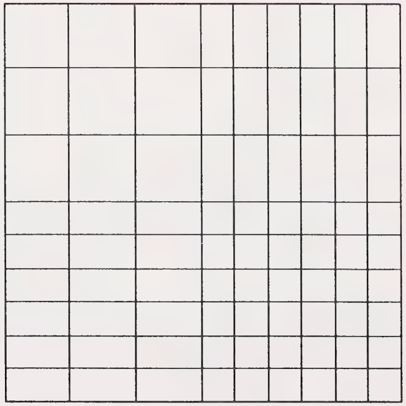


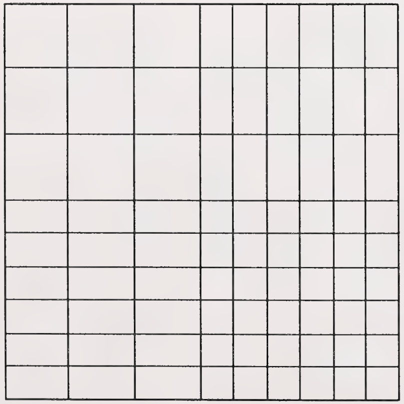


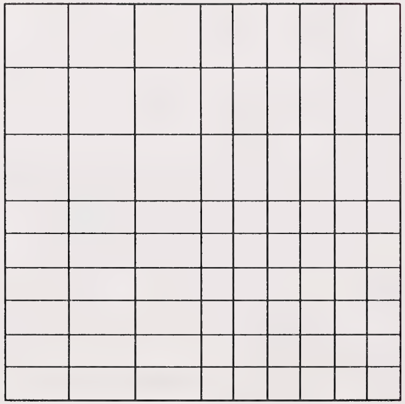


Binomial Grids

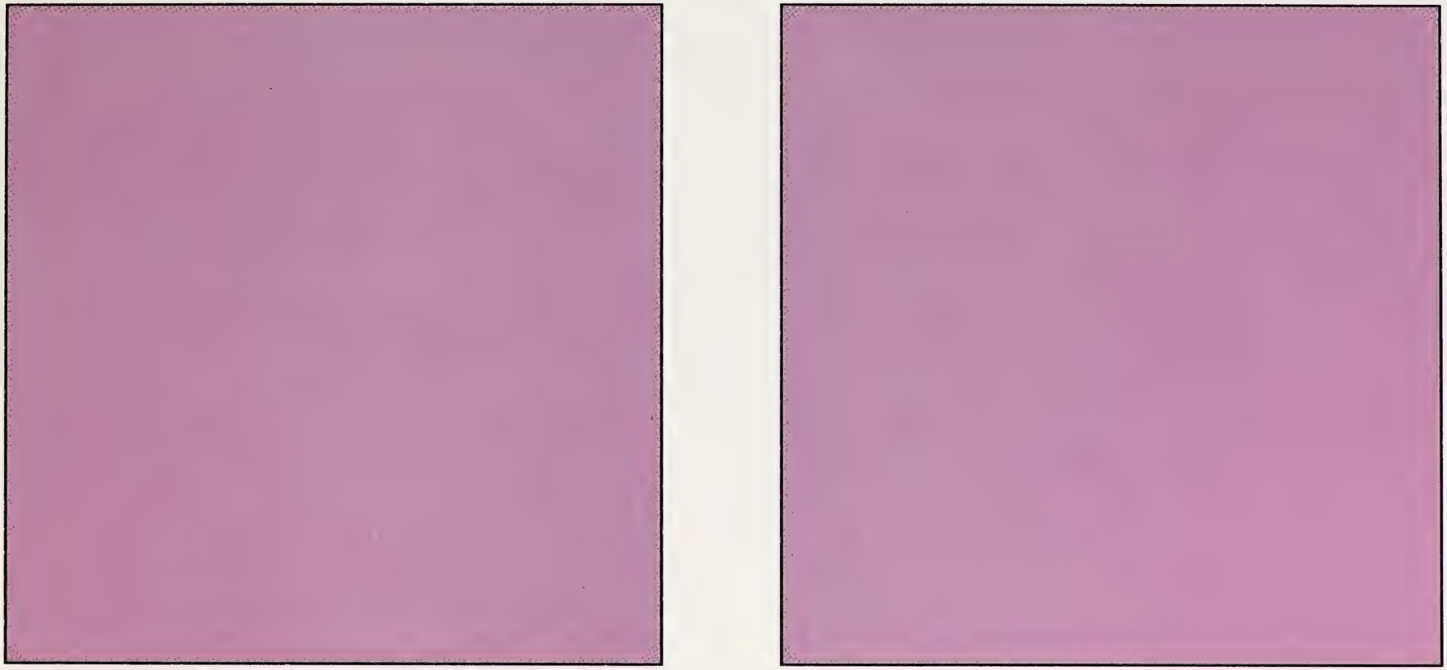








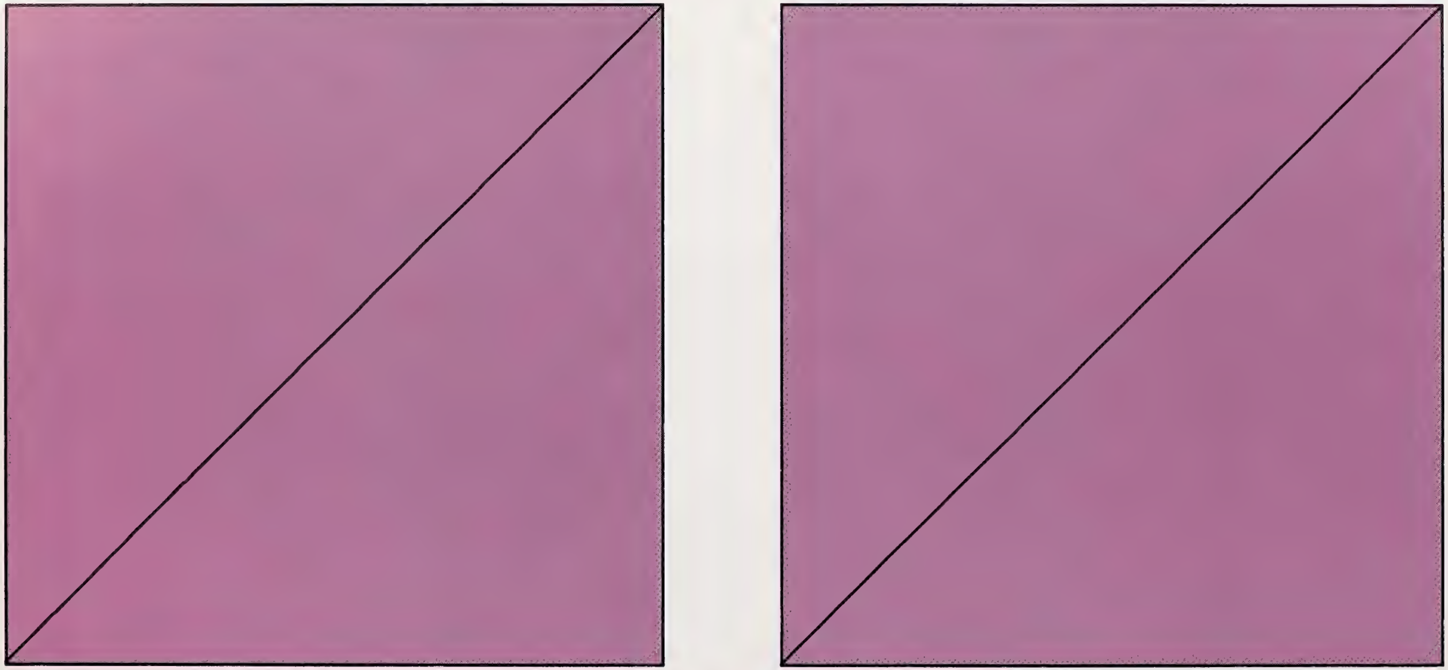
Polynomial Tiles



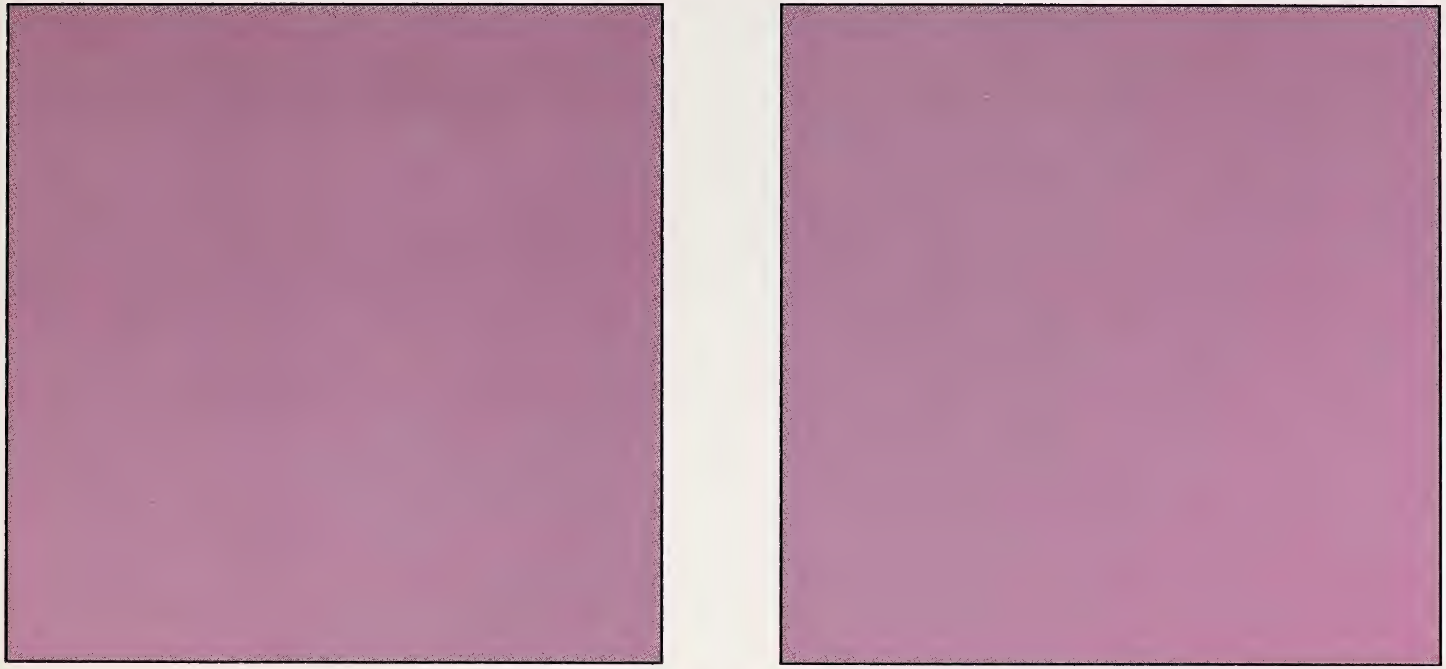
Each of these squares represents $+x^2$. Each one is called a flat.

Polynomial Tiles

Each of these squares represents $-x^2$. Each one is called a negative flat.



Polynomial Tiles



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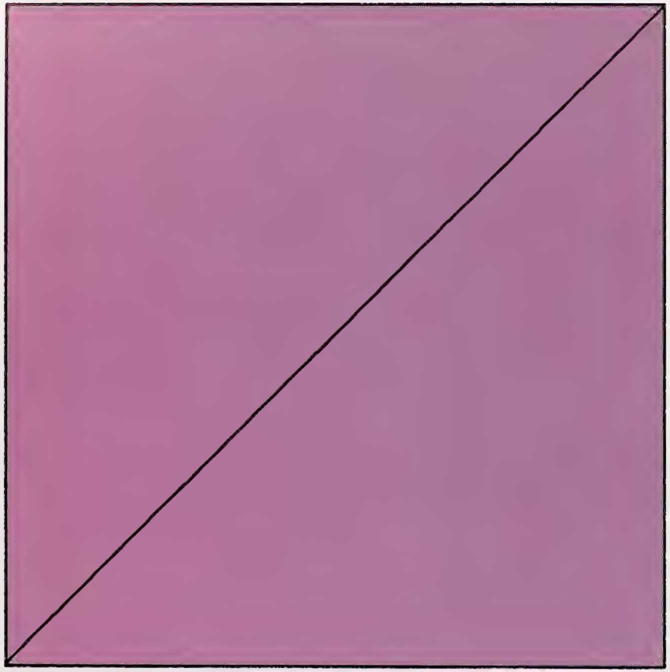
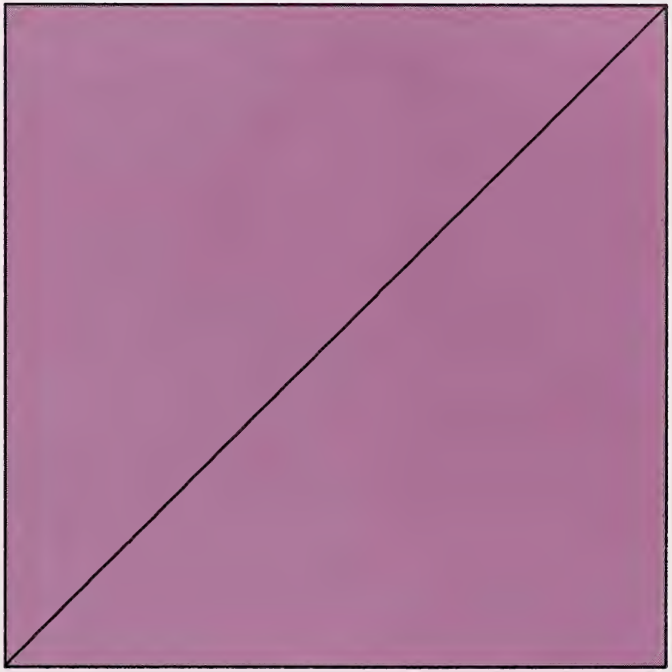
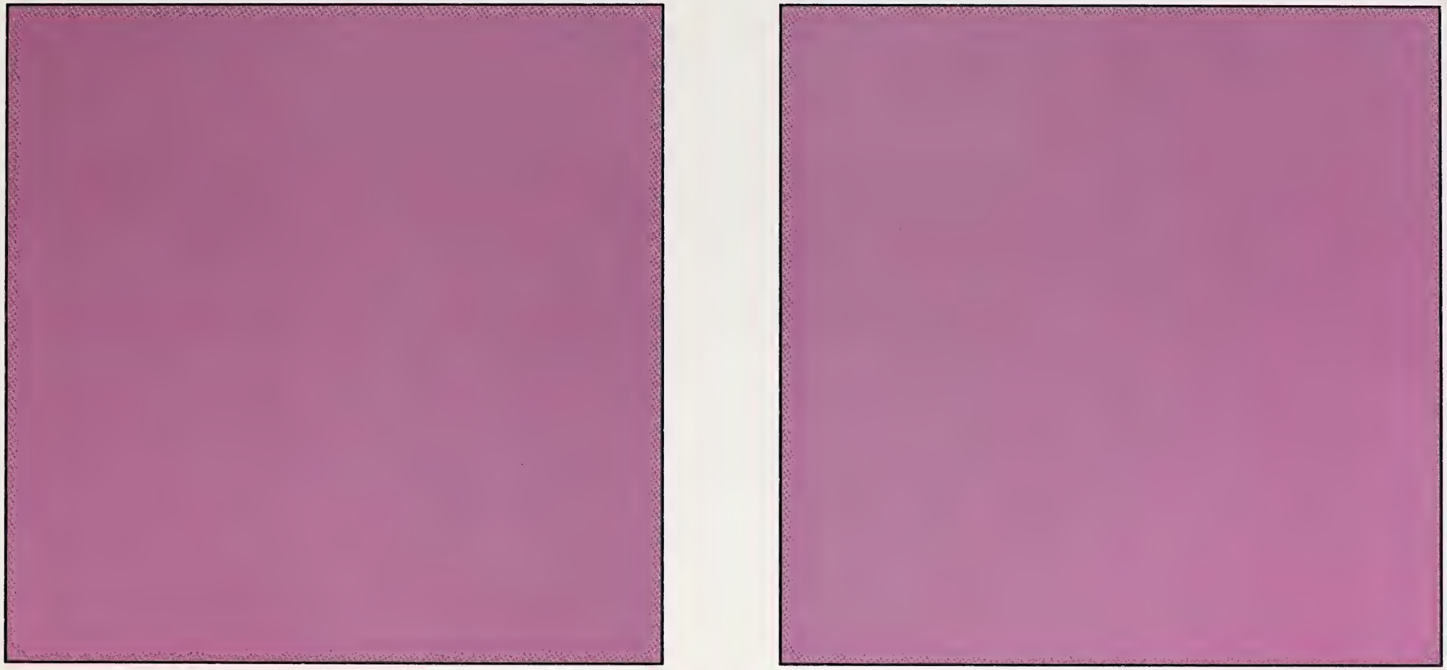




Polynomial Tiles



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Polynomial Tiles

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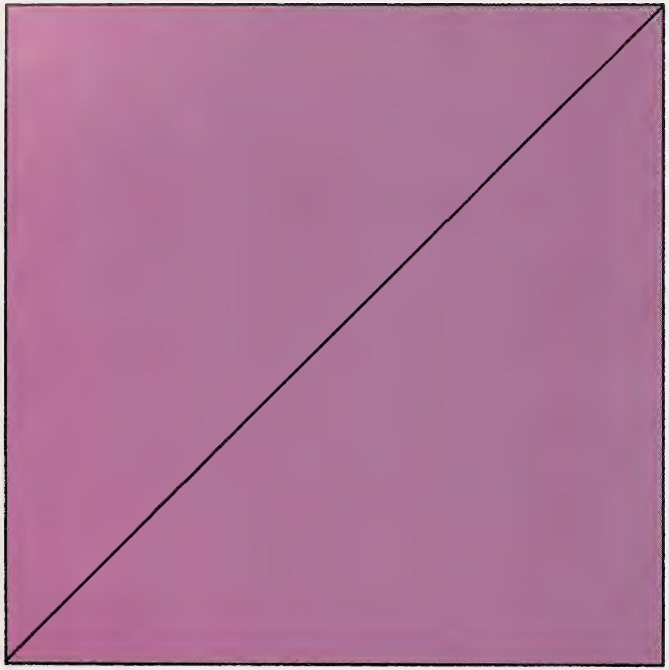
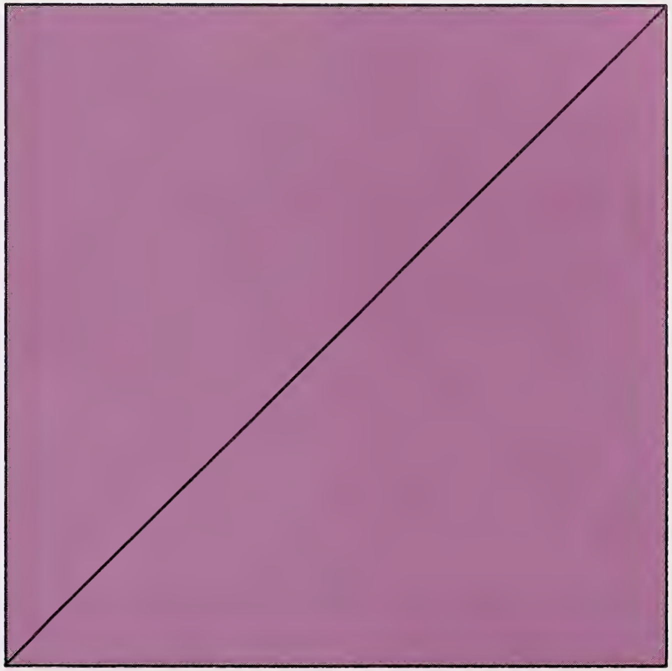
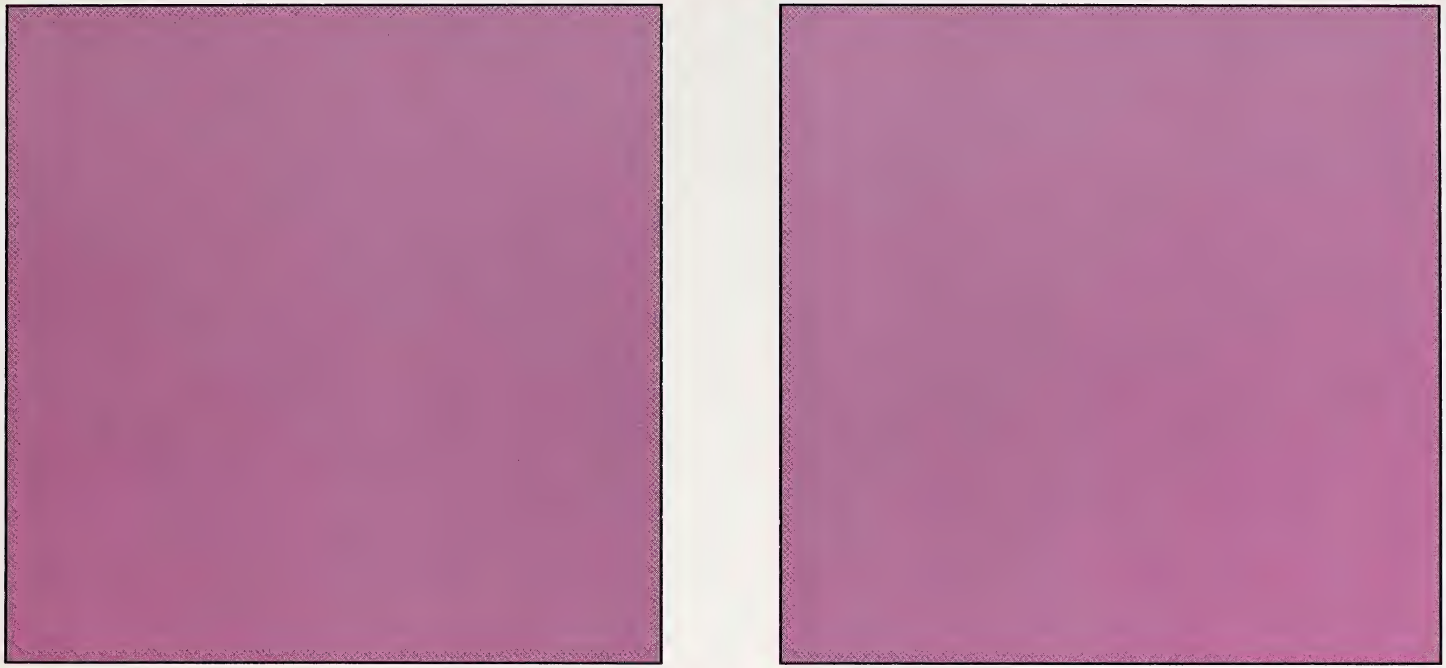




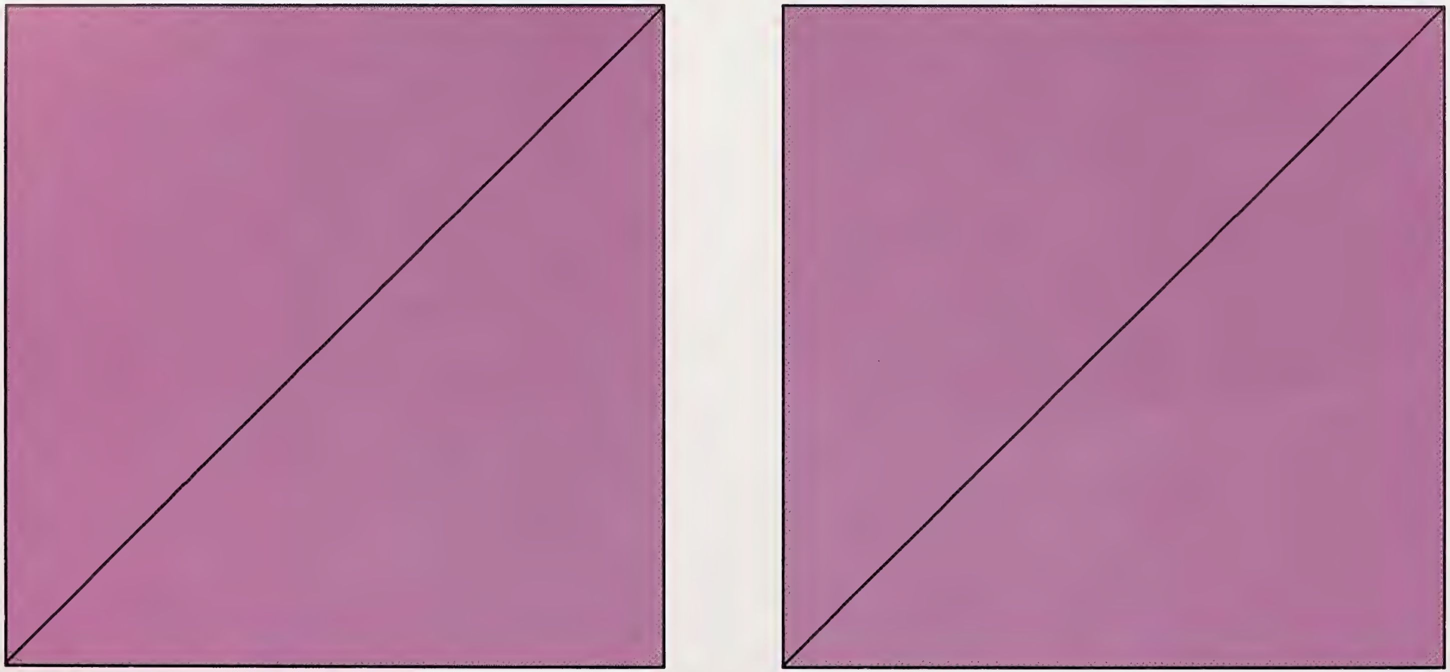
Polynomial Tiles



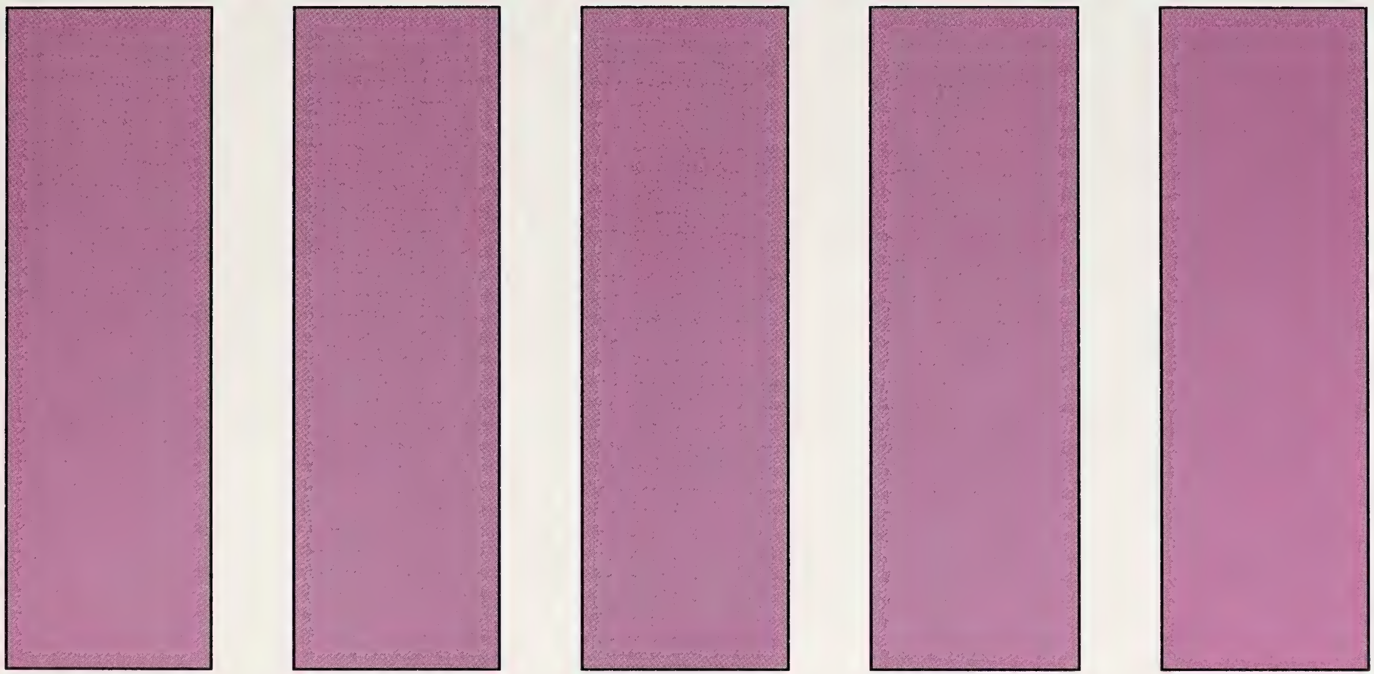
Each of these squares represents $+x^2$. Each one is called a flat.

Polynomial Tiles

Each of these squares represents $-x^2$. Each one is called a negative flat.



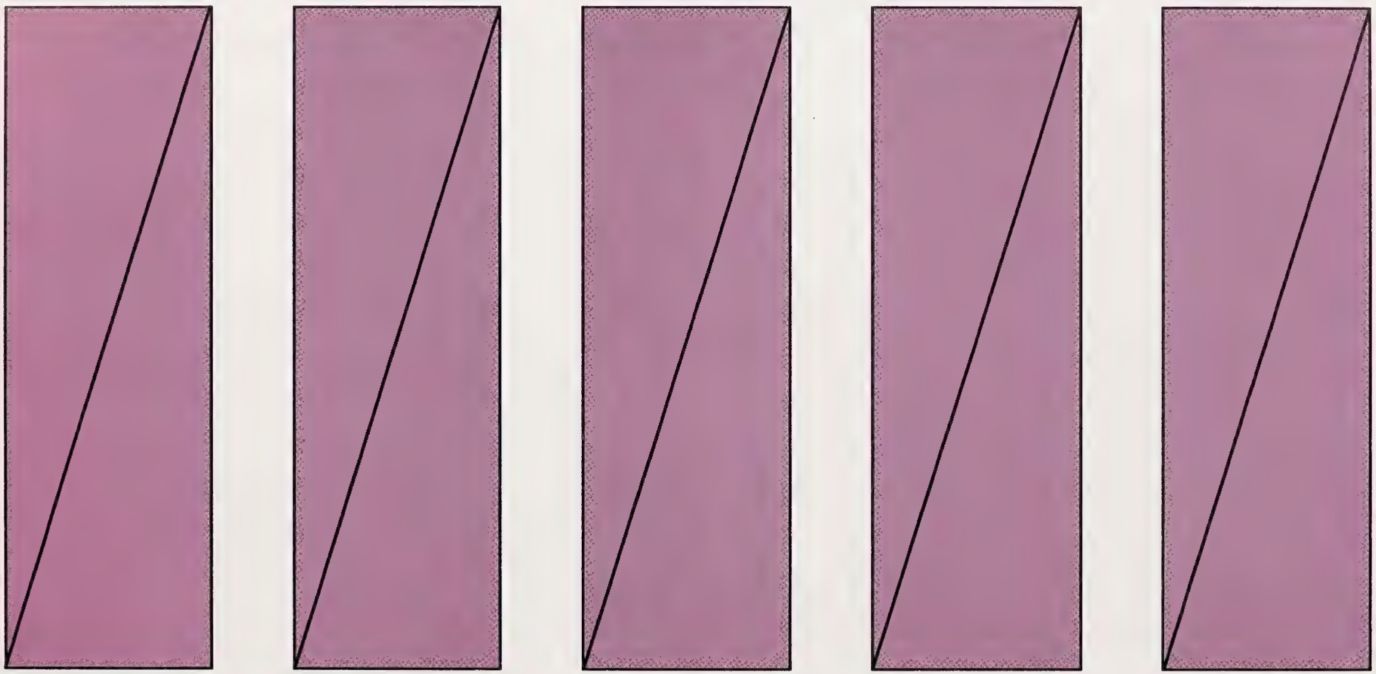
Polynomial Tiles



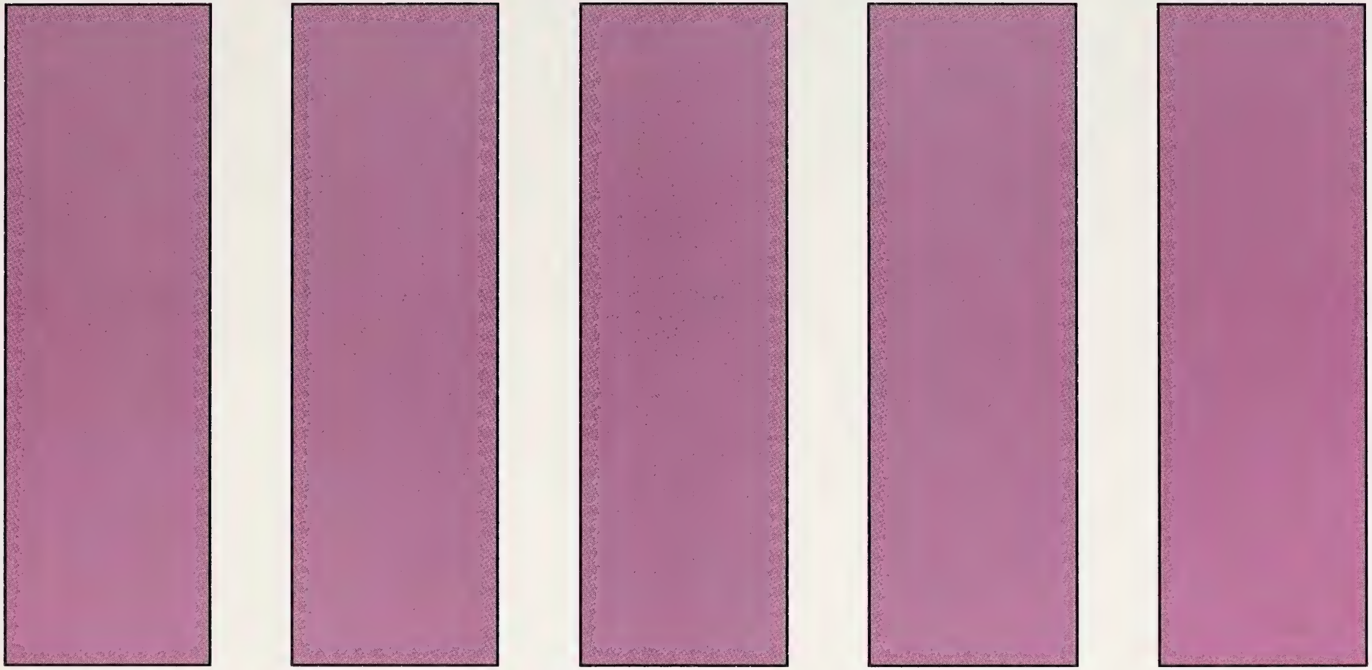
Each of these strips represents x. Each one is called a strip.

Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



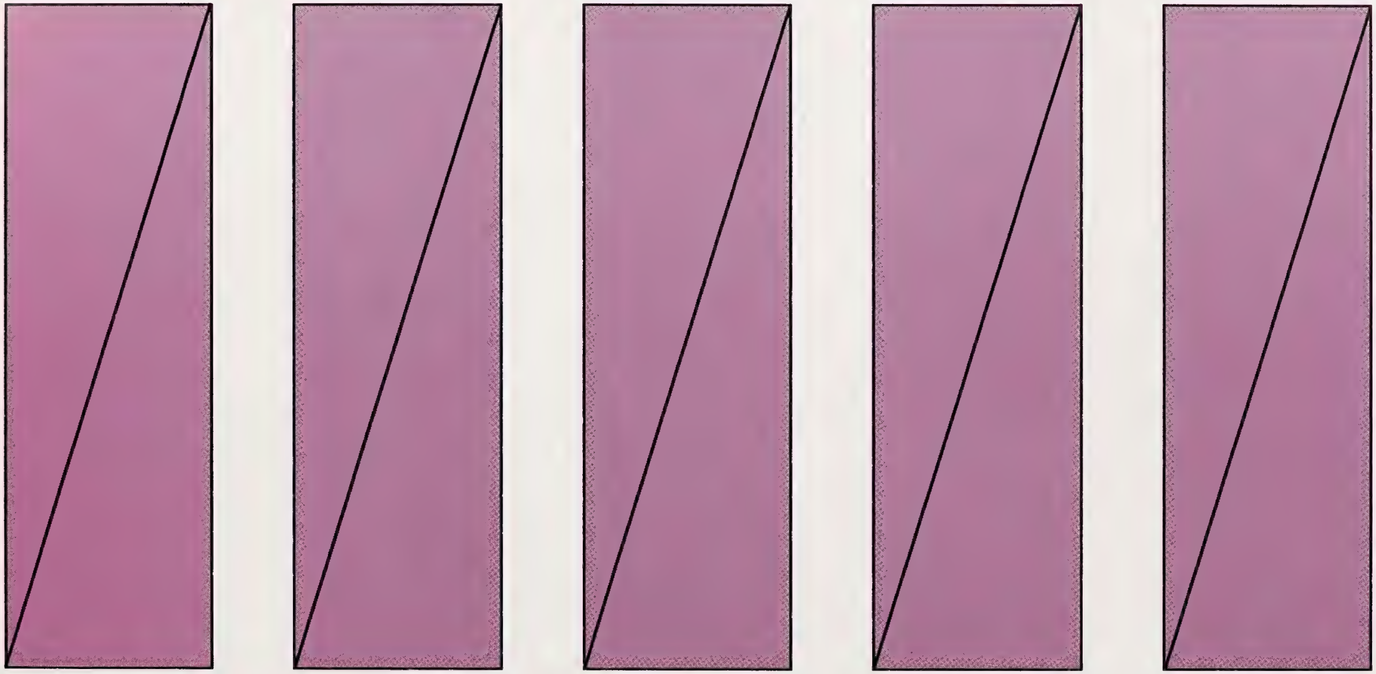
Polynomial Tiles



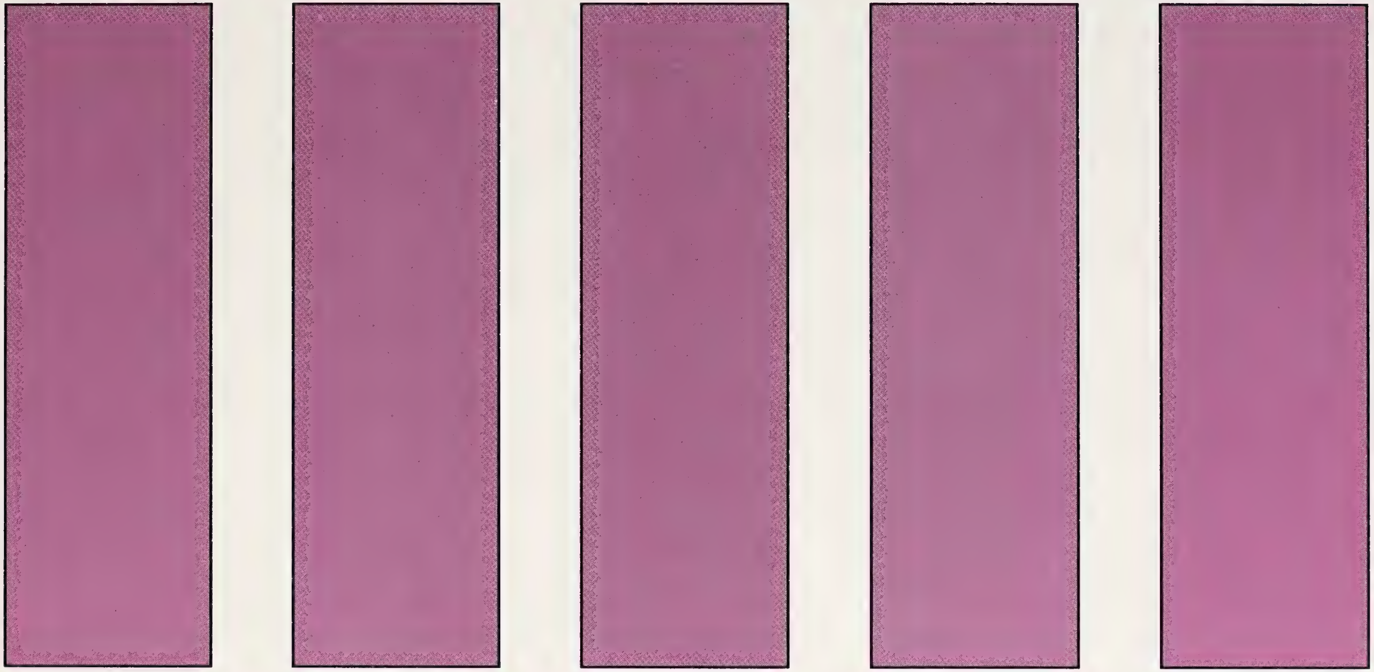
Each of these strips represents x. Each one is called a strip.

Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



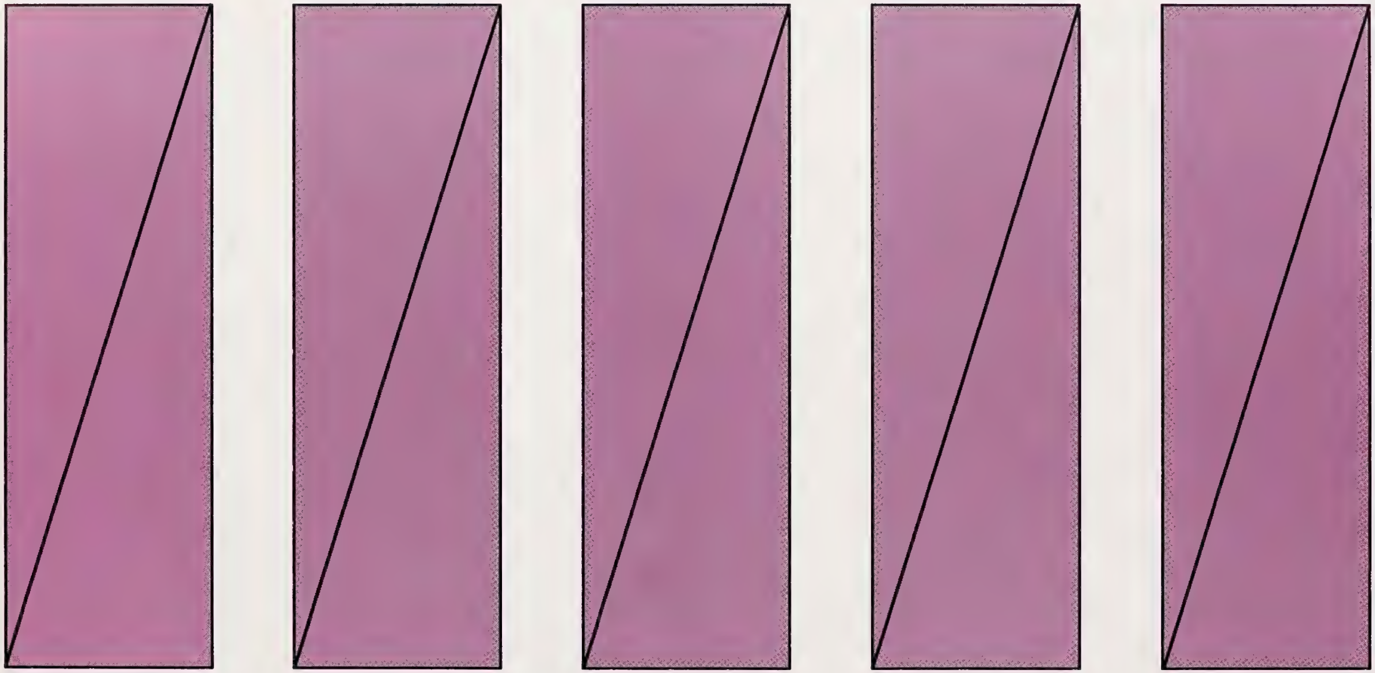
Polynomial Tiles



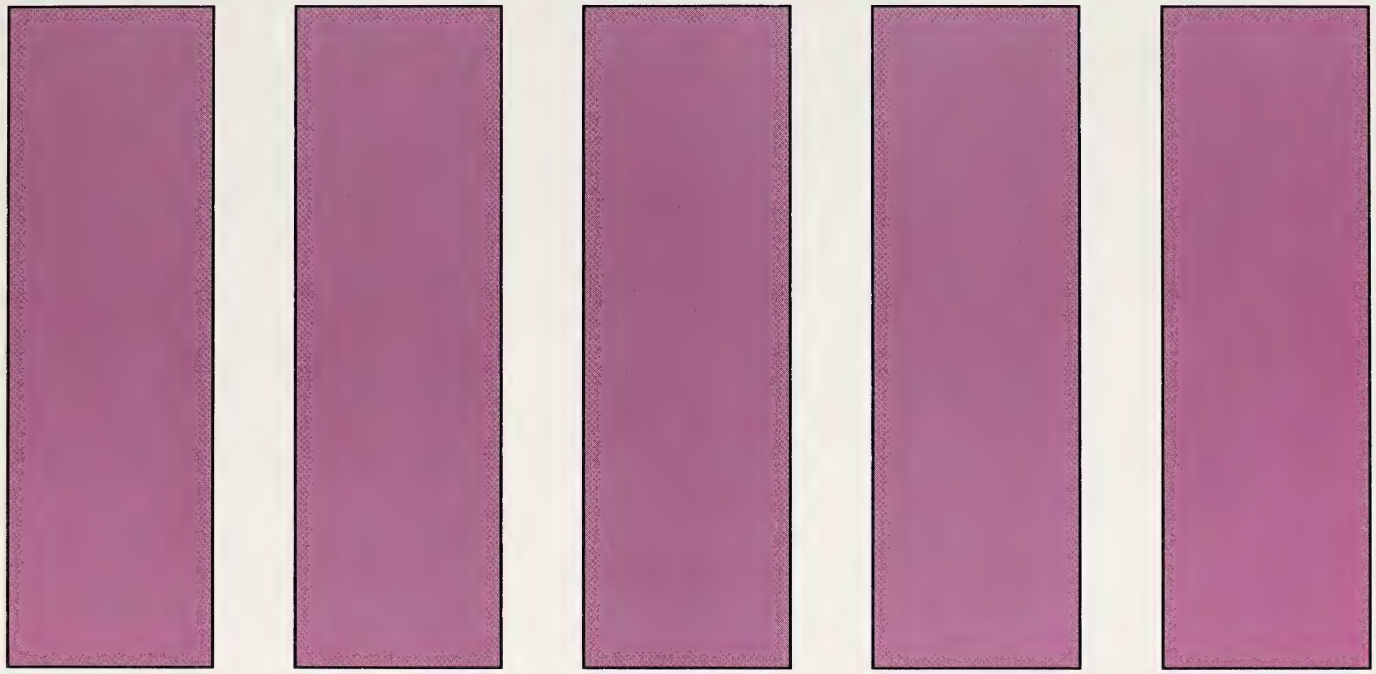
Each of these strips represents x. Each one is called a strip.

Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



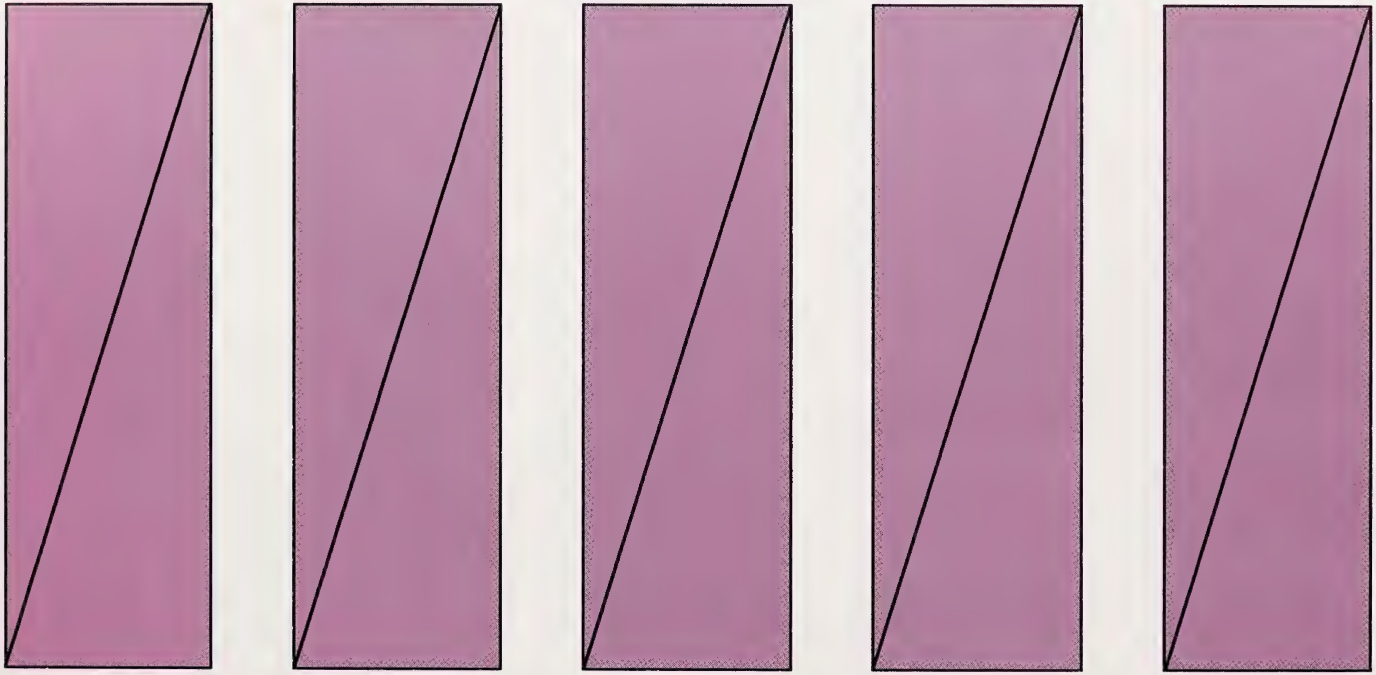
Polynomial Tiles



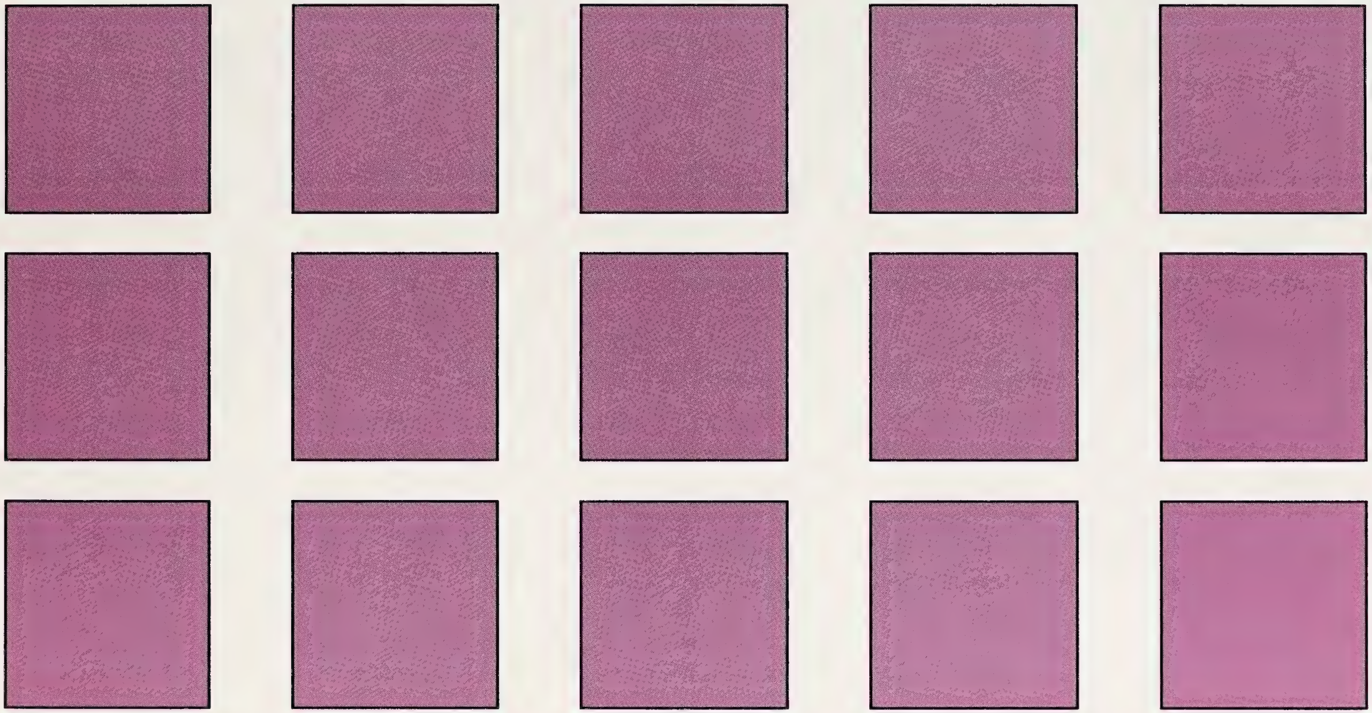
Each of these strips represents x. Each one is called a strip.

Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



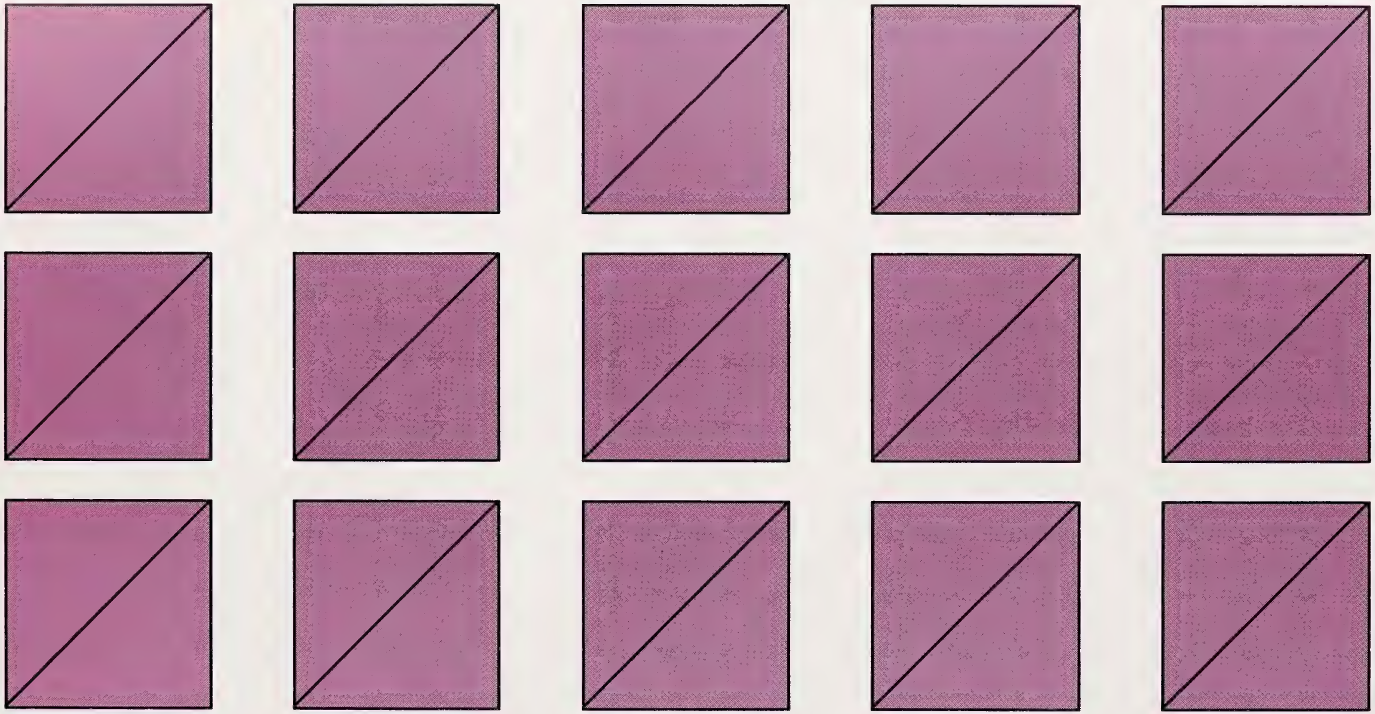
Polynomial Tiles



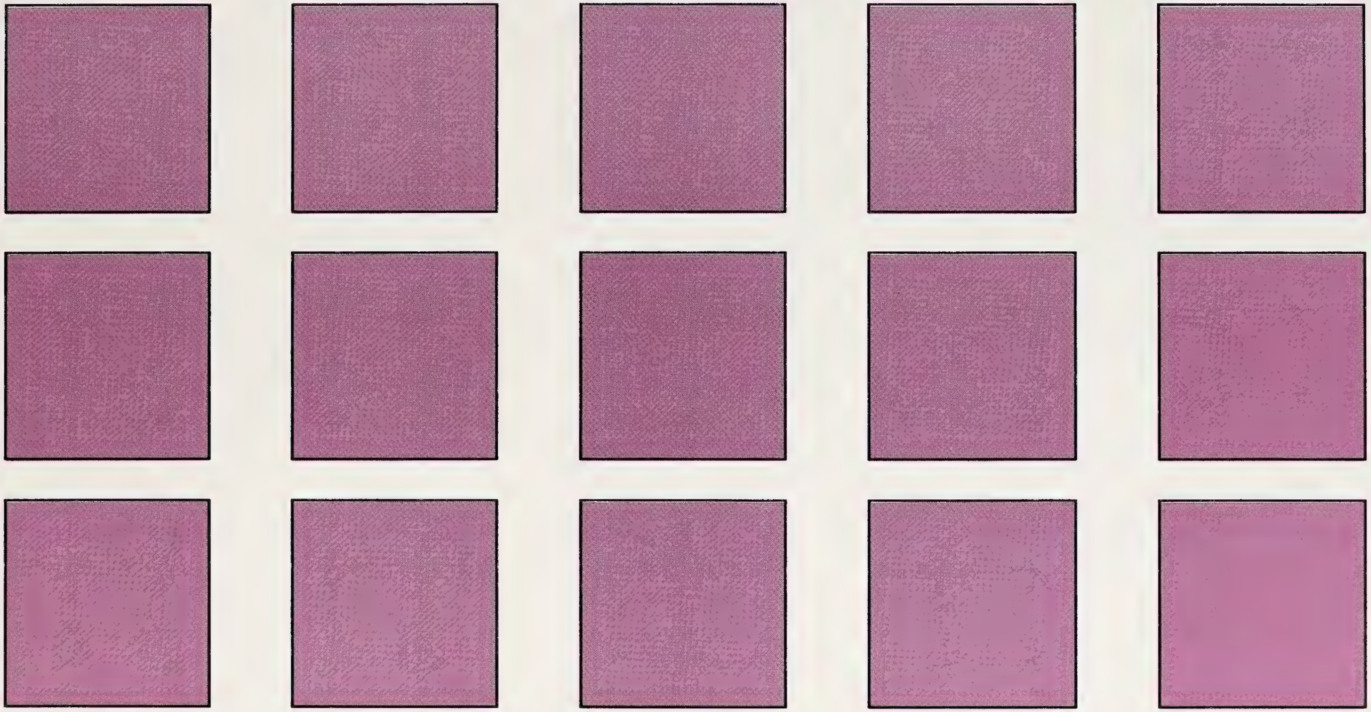
Each of these small squares represents 1. Each one is called a unit.

Polynomial Tiles

Each of these small squares represents -1. Each one is called a negative unit.



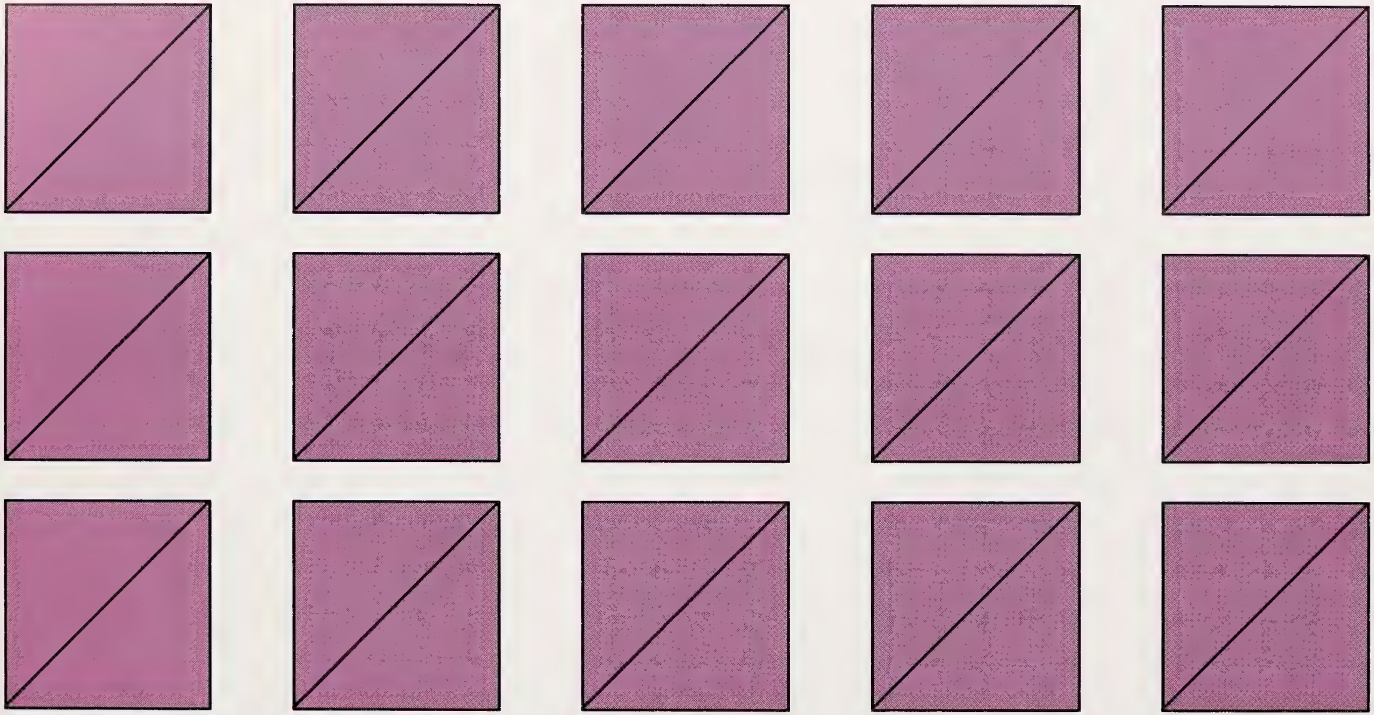
Polynomial Tiles



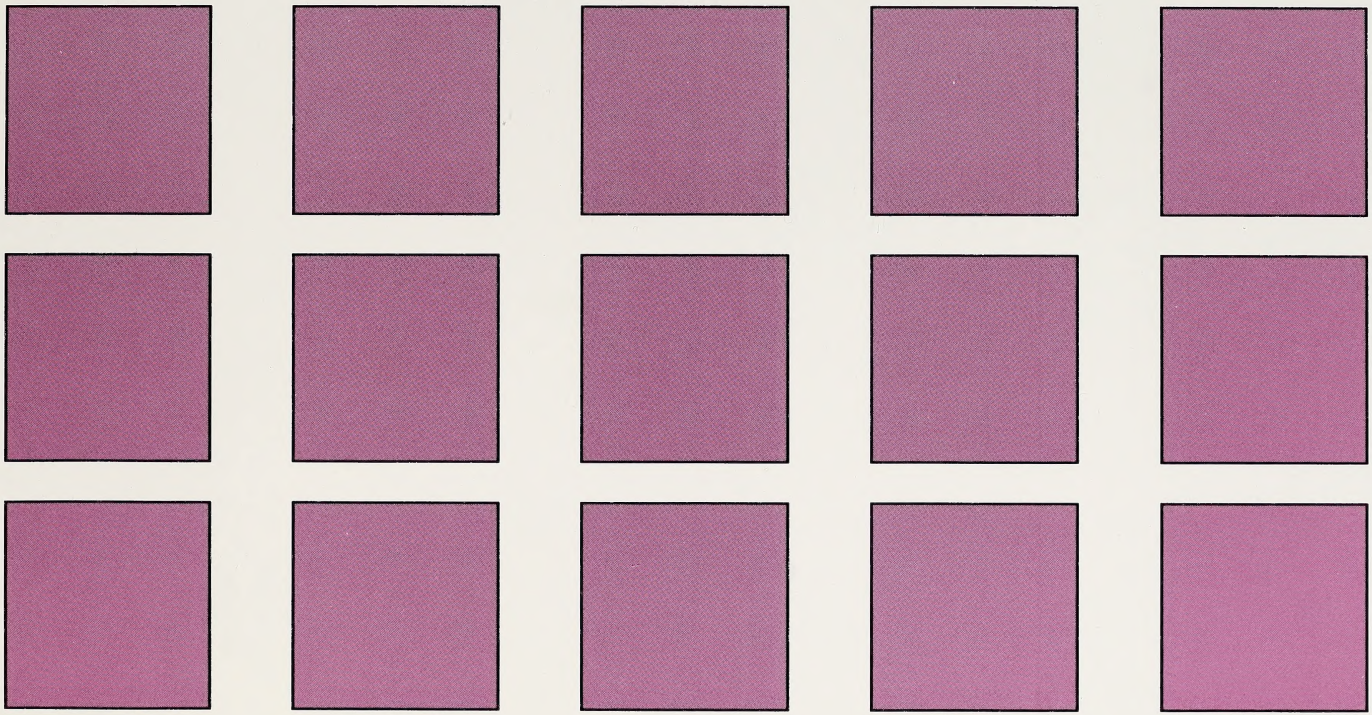
Each of these small squares represents 1. Each one is called a unit.

Polynomial Tiles

Each of these small squares represents -1. Each one is called a negative unit.



Polynomial Tiles



Each of these small squares represents 1. Each one is called a unit.

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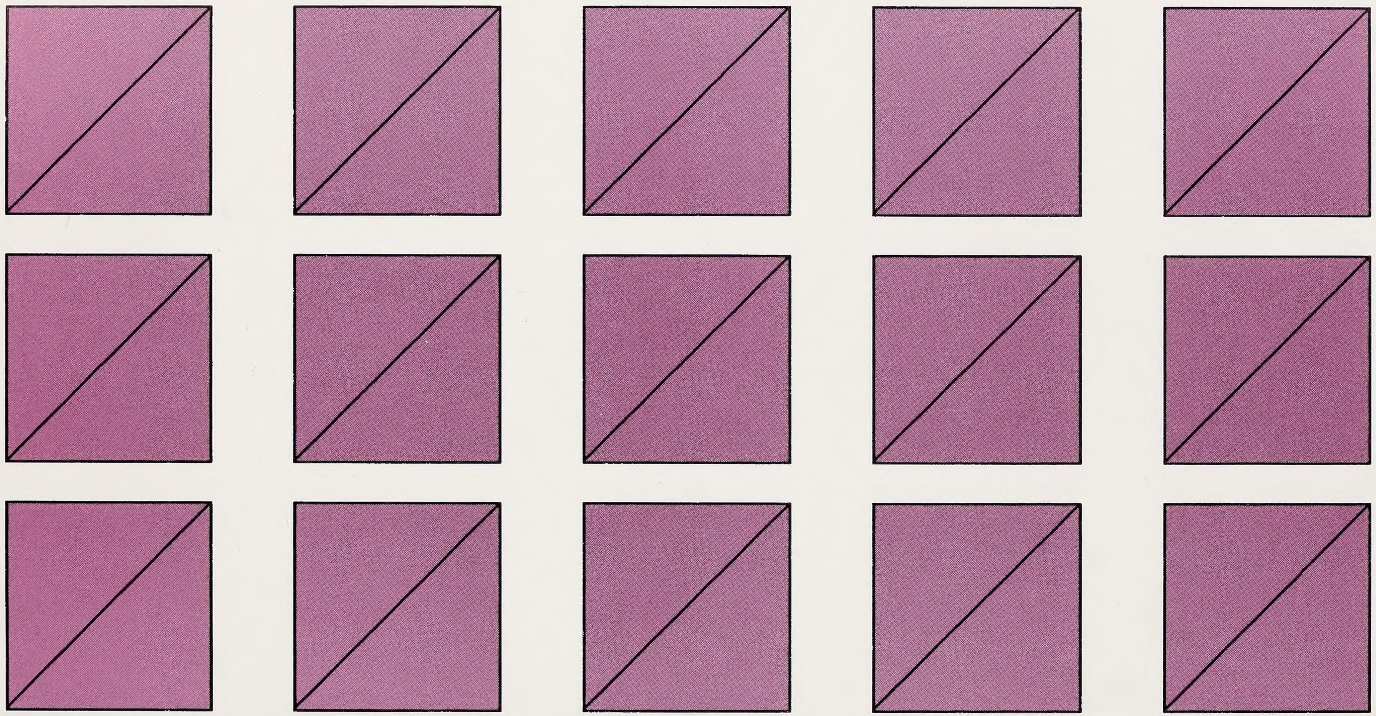

Mathematics 10

Student Module
Unit 2

L.R.D.C.
Producer

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